Biyani's Think Tank

***Concept based notes***

# Mathematical Methods for Numerical Analysis and Optimization

*(BCA Part-II)*



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# Preface

I am glad to present this book, especially designed to serve the needs of the students. The book has been written keeping in mind the general weakness in understanding the fundamental concepts of the topics. The book is self-explanatory and adopts the “Teach Yourself” style. It is based on question-answer pattern. The language of book is quite easy and understandable based on scientific approach.

This book covers basic concepts related to the microbial understandings about diversity, structure, economic aspects, bacterial and viral reproduction etc.

Any further improvement in the contents of the book by making corrections, omission and inclusion is keen to be achieved based on suggestions from the readers for which the author shall be obliged.

I acknowledge special thanks to Mr. Rajeev Biyani, *Chairman* & Dr. Sanjay Biyani, *Director* (*Acad.*) Biyani Group of Colleges, who are the backbones and main concept provider and also have been constant source of motivation throughout this Endeavour. They played an active role in coordinating the various stages of this Endeavour and spearheaded the publishing work.

I look forward to receiving valuable suggestions from professors of various educational institutions, other faculty members and students for improvement of the quality of the book. The reader may feel free to send in their comments and suggestions to the under mentioned address.

**Author**

# Syllabus

## B.C.A. Part-II

## Mathematical Methods for

## Numerical Analysis and Optimization

Computer arithmetics and errors. Algorithms and programming for numerical solutions. The impact of parallel computer : introduction to parallel architectures. Basic algorithms Iterative solutions of nonlinear equations : bisection method, Newton-Raphson method, the Secant method, the method of successive approximation. Solutions of simultaneous algebraic equations, the Gauss elimination method. Gauss-Seidel Method, Polynomial interpolation and other interpolation functions, spline interpolation system of linear equations, partial pivoting, matrix factorization methods. Numerical calculus : numerical differentiating, interpolatory quadrature. Gaussian integration. Numerical solutions of differential equations. Euler's method. Runge-Kutta method. Multistep method. Boundary value problems : shooting method.

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# Content

| S.No. | Name of Topic |
| --- | --- |
| 16. | Newton's Formula for Backward Interpolation |
| 17. | Divided Difference Interpolation |
| 18. | Lagrange's Interpolation |
| 19. | Spline Interpolation |
| 20. | Quadratic Splines |
| 21. | Cubic Splines |
| 22. | Numerical Differentiation |
| 23. | Numerical Integration |
| 24. | Euler's Method |
| 25. | Euler's Modified Method |
| 26. | Rungs – Kutta Method |
| 27. | Shooting Method |
| 28. | Unsolved Papers 2011 to 2006 |

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# Chapter-1

# Computer Arithmetic and Errors

**Q.1. An approximate value of $\pi$ is given by $x_1$ = 22/7 = 3.1428571 and its true value is x = 3.1415926. Find the absolute and relative errors.**

**Ans.:** True value of $\pi(x) = 3.1415926$

Approximate value of $\pi(x_1) = 3.1428571$

**Absolute error** is given by –

$$E_a = |x - x_1|$$

$$= |3.1415926 - 3.1428571|$$

$$= 0.0012645$$

**Relative error** is given by –

$$E_r = \left|\frac{x - x_1}{x}\right|$$

$$= \left|\frac{3.1415926 - 3.1428571}{3.1415926}\right|$$

$$= \left|\frac{0.0012645}{3.1415926}\right|$$

$$= 0.0004025$$

**Q.2. Let x = 0.00458529 find the absolute error if x is truncated to three decimal digits.**

**Ans.:** $x = 0.00458529 = 0.458529 \times 10^{-2}$ [in normalized floating point form]

$x_1 = 0.458 \times 10^{-2}$ [after truncating to three decimal places]

Absolute error $= |x - x_1|$

$= |0.458529 \times 10^{-2} - 0.458 \times 10^{-2}|$

$= 0.000529 \times 10^{-2}$

$= 0.000529 \text{ E} - 2$

$= 0.529 \text{ E} - 5$

**Q.3. Let the solution of a problem be $x_a = 35.25$ with relative error in the solution atmost 2% find the range of values upto 4 decimal digits, within which the exact value of the solution must lie.**

**Ans.:** We are given that the approximate solution of the problem is $(x_a) = 35.25$ and it has relative error upto 2% so

$$\left|\frac{x - 35.25}{x}\right| < 0.02$$

$$= -0.02 < \frac{x - 35.25}{x} < 0.02$$

**Case-I :** if $-0.02x < \frac{x - 35.25}{x}$

$\Rightarrow -0.02x < x - 35.25$

$\Rightarrow 35.25 < x + 0.02x$

$\Rightarrow 35.25 < x (1 + 0.02)$

$\Rightarrow 35.25 < x (1.02)$

$\Rightarrow 35.25 < 1.02x$

$\Rightarrow \frac{35.25}{1.02} < x$

$\Rightarrow x > 34.5588$ _ _ _ (1)

**Case-II**: if $\frac{x - 35.25}{x} < 0.02$

$\Rightarrow x - 35.25 < 0.02x$

$\Rightarrow x - 0.02x < 35.25$

$\Rightarrow 0.98x < 35.25$

$\Rightarrow x < \frac{35.25}{0.98}$

$\Rightarrow x < 35.9693$ _ _ _ (2)

From equation (1) and (2) we have $34.5588 < x < 35.9693$

$\therefore$ The required range is (34.5588, 35.9693)

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## Chapter-2

# Bisection Method

**Q.1. Find real root of the equation $x^3$ - 5x + 3 upto three decimal digits.**

**Ans.:** Here $f(x) = x^3 - 5x + 3$

$f(0) = 0 - 0 + 3 = 3\ f(x_0)$ (say)

$f(1) = 1 - 5 + 3 = -1 = f(x_1)$ (say)

Since $f(x_0), f(x_1) < 0$ so the root of the given equation lies between 0 and 1

So, $x_2 = \frac{x_0 + x_1}{2} = \frac{0+1}{2} = 0.5$

Now, $f(x_2) = f(0.5)$

$= (0.5)^3 - 5\,(0.5) + 3$

$= 0.125 - 2.5 + 3$

$= 0.625$ (which is positive)

$\because \quad f(x_1).f(x_2) < 0$

So, $x_3 = \frac{x_1 + x_2}{2} = \frac{1 + 0.5}{2} = 0.75$

Now, $f(x_3) = f(0.75)$

$= (0.75)^3 - 5\,(0.75) + 3$

$= 0.4218 - 3.75 + 3$

$= -\,0.328$ (which is negative)

$\because \quad f(x_2).f(x_3) < 0$

So, $x_4 = \frac{x_2 + x_3}{2} = \frac{0.5 + 0.75}{2} = 0.625$

Now, $f(x_4) = f(0.625)$

$= (0.625)^3 - 5\,(0.625) + 3$

$= 0.244 - 3.125 + 3$

$= 0.119$ (which is positive)

$\because \quad f(x_3).f(x_4) < 0$

So, $x_5 = \frac{x_3 + x_4}{2} = \frac{0.75 + 0.625}{2} = 0.687$

Now, $f(x_5) = f(0.687)$

$= (0.687)^3 - 5\,(0.687) + 3$

$= -\,0.1108$ (which is negative)

$\because \quad f(x_4).f(x_5) < 0$

So, $x_6 = \frac{x_4 + x_5}{2} = \frac{0.625 + 0.687}{2} = 0.656$

Now, $f(x_6) = f(0.656)$

$= (0.656)^3 - 5\,(0.656) + 3$

$= 0.0023$ (which is positive)

$\because \quad f(x_5).f(x_6) < 0$

So, $x_7 = \frac{x_5 + x_6}{2} = \frac{0.687 + 0.656}{2} = 0.671$

Now, $f(x_7) = f(0.671)$

$= (0.671)^3 - 5\,(0.671) + 3$

$= -\,0.0528$ (which is negative)

$\because \quad f(x_6).f(x_7) < 0$

So, $x_8 = \frac{x_6 + x_7}{2} = \frac{0.656 + 0.671}{2} = 0.663$

Now, $f(x_8) = f(0.663)$

$= (0.663)^3 – 5 (0.663) + 3$

$= 0.2920 – 3.315 + 3$

$= – 0.023$ (which is negative)

$\because \quad f(x_6).f(x_8) < 0$

So, $x_9 = \frac{x_6 + x_8}{2} = \frac{0.656 + 0.663}{2} = 0.659$

Now, $f(x_9) = f(0.659)$

$= (0.659)^3 – 5 (0.659) + 3$

$= – 0.0089$ (which is negative)

$\because \quad f(x_6).f(x_9) < 0$

So, $x_{10} = \frac{x_6 + x_9}{2} = \frac{0.656 + 0.659}{2} = 0.657$

Now, $f(x_{10}) = f(0.657)$

$= (0.657)^3 – 5 (0.657) + 3$

$= – 0.00140$ (which is negative)

$\because \quad f(x_6).f(x_{10}) < 0$

So, $x_{11} = \frac{x_6 + x_{10}}{2} = \frac{0.656 + 0.657}{2} = 0.656$

Now, $f(x_{11}) = f(0.656)$

$= (0.656)^3 – 5 (0.656) + 3$

$= 0.2823 – 3.28 + 3$

$= 0.00230$ (which is positive)

$\because \quad f(x_{11}).f(x_{10}) < 0$

So, $x_{12} = \frac{x_{10} + x_{11}}{2} = \frac{0.657 + 0.656}{2} = 0.656$

Since $x_{11}$ and $x_{12}$ both same value. Therefore if we continue this process we will get same value of x so the value of x is 0.565 which is required result.

**Q.2. Find real root of the equation cos x - xe$^x$ = 0 correct upto four decimal places.**

**Ans.:** Since, $f(x) = \cos x - xe^x$

So, $f(0) = \cos 0 - 0e^0 = 1$ (which is positive)

And $f(1) = \cos 1 - 1e^1 = -2.1779$ (which is negative)

$\because$ $f(0).f(1) < 0$

Hence the root of are given equation lies between 0 and 1.

let $f(0) = f(x_0)$ and $f(1) = f(x_1)$

So, $x_2 = \dfrac{x_0 + x_1}{2} = \dfrac{0 + 1}{2} = 0.5$

Now, $f(x_2) = f(0.5)$

$f(0.5) = \cos(0.5) - (0.5)e^{0.5}$

$= 0.05322$ (which is positive)

$\because$ $f(x_1).f(x_2) < 0$

So, $x_3 = \dfrac{x_1 + x_2}{2} = \dfrac{1 + 0.5}{2} = \dfrac{1.5}{2} = 0.75$

Now, $f(x_3) = f(0.75)$

$= \cos(0.75) - (0.75)e^{0.75}$

$= -0.856$ (which is negative)

$\because$ $f(x_2).f(x_3) < 0$

So, $x_4 = \dfrac{x_2 + x_3}{2} = \dfrac{0.5 + 0.75}{2} = 0.625$

$f(x_4) = f(0.625)$

$= \cos(0.625) - (0.625)e^{(0.625)}$

$= -0.356$ (which is negative)

$\because$ $f(x_2).f(x_4) < 0$

So, $x_5 = \dfrac{x_2 + x_4}{2} = \dfrac{0.5 + 0.625}{2} = 0.5625$

Now, $f(x_3) = f(0.5625)$

$= \cos(0.5625) - 0.5625e^{0.5625}$

$= -0.14129$ (which is negative)

$\because \quad f(x_2).f(x_5) < 0$

So, $x_6 = \dfrac{x_2 + x_5}{2} = \dfrac{0.5 + 0.5625}{2} = 0.5312$

Now, $f(x_6) = f(0.5312)$

$= \cos(0.5312) - (0.5312)e^{0.5312}$

$= -0.0415$ (which is negative)

$\because \quad f(x_2).f(x_6) < 0$

So, $x_7 = \dfrac{x_2 + x_6}{2} = \dfrac{0.5 + 0.5312}{2} = 0.5156$

Now, $f(x_7) = f(0.5156)$

$= \cos(0.5156) - (0.5156)e^{0.5156}$

$= 0.006551$ (which is positive)

$\because \quad f(x_6).f(x_7) < 0$

So, $x_8 = \dfrac{x_6 + x_7}{2} = \dfrac{0.513 + 0.515}{2} = 0.523$

Now, $f(x_8) = f(0.523)$

$= \cos(0.523) - (0.523)e^{0.523}$

$= -0.01724$ (which is negative)

$\because \quad f(x_7).f(x_8) < 0$

So, $(x_9) = \dfrac{x_7 + x_8}{2} = \dfrac{0.515 + 0.523}{2} = 0.519$

Now, $f(x_9) = f(0.519)$

$= \cos(0.519) - (0.519)e^{0.519}$

$= -0.00531$ (which is negative)

$\because \quad f(x_7).f(x_9) < 0$

So, $(x_{10}) = \dfrac{x_7 + x_9}{2} = \dfrac{0.515 + 0.519}{2} = 0.5175$

Now, $f(x_{10}) = f(0.5175)$

$= \cos(0.5175) - (0.5175)e^{0.5175}$

$= 0.0006307$ (which is positive)

$\because \quad f(x_9).f(x_{10}) < 0$

So, $x_{11} = \dfrac{x_9 + x_{10}}{2} = \dfrac{0.5195 + 0.5175}{2} = 0.5185$

Now, $f(x_{11}) = f(0.5185)$

$= \cos(0.5185) - (0.5185)e^{0.5185}$

$= -0.002260$ (which is negative)

$\because \quad f(x_{10}).f(x_{11}) < 0$

So, $x_{12} = \dfrac{x_{10} + x_{11}}{2} = \dfrac{0.5175 + 0.5185}{2} = 0.5180$

Hence the root of the given equation upto 3 decimal places is x = 0.518

Thus the root of the given equation is x = 0.518

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## Chapter-3

# Regula Falsi Method

**Q.1. Find the real root of the equation $x \log_{10} x - 1.2 = 0$ correct upto four decimal places.**

**Ans.:** Given $f(x) = x \log_{10} x - 1.2$ _ _ _ (1)

In this method following formula is used -

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})\ f(x_n)}{(f(x_n) - f(x_{n-1}))} \quad \text{\_ \_ \_ (2)}$$

Taking x = 1 in eq.(1)

$f(1) = 1. \log_{10}1 - 1.2$

$= -2$ (which is negative)

Taking x = 2 in eq.(1)

$f(2) = 2. \log_{10} 2 - 1.2$

$= -0.5979$ (which is negative)

Taking x = 3 in eq.(1)

$f(3) = 3. \log_{10} 3 - 1.2$

$= 0.2313$ (which is positive)

$\because \quad f(2).f(3) < 0$

So the root of the given equation lies between 2 and 3.

let $x_1 = 2$ and $x_2 = 3$

$\therefore \quad f(x_1) = f(2) = -0.5979$

And $f(x_2) = f(3) = 0.2313$

Now we want to find $x_3$ so using eq.(2)

$$x_3 = x_2 - \frac{(x_2 - x_1)\ f(x_2)}{f(x_2) - f(x_1)}$$

$$= 3 - \frac{(3-2) \times (0.2313)}{0.2313 - (-0.5979)}$$

$$= 3 - \frac{0.2313}{0.8292}$$

$$= 3 - 0.2789 = 2.7211$$

$$f(x_3) = f(2.7211)$$

$$= 2.7211 \log_{10} 2.7211 - 1.2$$

$$= -0.01701 \text{ (which is negative)}$$

$\because \quad f(x_2).f(x_3) < 0$

Now to find $x_4$ using equation (2)

$$x_4 = x_3 - \frac{(x_3 - x_2)\ f(x_3)}{f(x_3) - f(x_2)}$$

$$= 2.7211 - \frac{(2.7211 - 3) \times (-0.0170)}{(-0.0170 - 0.2313)}$$

$$= 2.7211 - \frac{0.004743}{0.2483}$$

$$= 2.7211 + 0.01910 = 2.7402$$

Now

$$f(x_4) = f(2.7402)$$

$$= 2.7402 \log_{10} 2.7402 - 1.2$$

$$= -0.0003890 \text{ (which is negative)}$$

$\because \quad f(x_2).f(x_4) < 0$

Now to find $x_5$ using equation (2)

$$x_5 = x_4 - \frac{(x_4 - x_2)\ f(x_4)}{[f(x_4) - f(x_2)]}$$

$$= 2.7402 - \frac{(2.7402 - 3)}{(-0.0004762 - 0.2313)} \times (-0.0004762)$$

$$= 2.7402 + \frac{(-0.2598)(-0.0004762)}{0.2317}$$

$$= 2.7402 + \frac{(0.0001237)}{0.2317}$$

$$= 2.7402 + 0.0005341 = 2.7406$$

$f(x_5) = f(2.7406)$

$= 2.7406 \log_{10} 2.7406 - 1.2$

$= -\ 0.0000402$ (which is negative)

$\because \quad f(x_2).f(x_5) < 0$

To find $x_6$ using equation (2)

$$x_6 = x_5 - \frac{(x_5 - x_2)\ f(x_5)}{f(x_5) - f(x_2)}$$

$$= 2.7406 + \frac{(2.7406 - 3) \times (-0.000040)}{(-0.00004) - (0.2313)}$$

$$= 2.7406 + 0.000010 = 2.7406$$

$\therefore$ The approximate root of the given equation is 2.7406 which is correct upto four decimals.

**Q.2. Find the real root of the equation $x^3 - 2x - 5 = 0$ correct upto four decimal places.**

**Ans.:** Given equation is

$f(x) = x^3 - 2x - 5$ _ _ _ (1)

In this method following formula is used :-

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})\ f(x_n)}{[f(x_n) - f(x_{n-1})]} \qquad \text{---} \ (2)$$

Taking x = 1 in equation (1)

$f(1) = 1 - 2 - 5 = -6$ (which is negative)

Taking x = 2 in equation (1)

$f(2) = 8 - 4 - 5 = -1$ (which is negative)

Taking x = 3

$f(3) = 27 - 6 - 5 = 16$ (which is positive)

Since $f(2).f(3) < 0$

So the root of the given equation lies between 2 and 3.

Let $x_1 = 2$ and $x_2 = 3$

$f(x_1) = f(2) = -1$

and $f(x_2) = f(3) = 16$

Now to find $x_3$ using equation (2)

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} f(x_2)$$

$$= 3 - \frac{(3 - 2)}{16 + 1} \times 16$$

$$= 3 - \frac{16}{17} = 2.0588$$

$f(x_3) = (2.0558)^3 - 2\,(2.0588) - 5$

$= 8.7265 - 4.1176 - 5$

$= -0.3911$ (which is negative)

$\therefore \quad f(x_2).f(x_3) < 0$

Now to find $x_4$ using equation (2)

$$x_4 = x_3 - \frac{(x_3 - x_2)}{[f(x_3) - f(x_2)]} \times f(x_3)$$

$$= 2.0588 - \frac{(2.0588 - 3)}{-0.3911 - 16} \times (-0.3911)$$

$$= 2.0588 + \frac{(-0.9412) \times (-0.3911)}{16.3911} = 2.0812$$

$\therefore$ $f(x_4) = 9.0144 - 4.1624 - 5$

$= -0.148$ (which is negative)

So $f(x_2) \cdot f(x_4) < 0$

Now using equation (2) to find $x_5$

$$x_5 = x_4 - \frac{(x_4 - x_2)}{[f(x_4) - f(x_2)]} \times f(x_4)$$

$$= 2.0812 - \frac{(2.0812 - 3)}{(-0.148 - 16)} \times (-0.148)$$

$$= 2.0812 + \frac{(-0.9188) \times (-0.148)}{16.148}$$

$$= 2.0812 + 8.4210 \times \frac{(x_5 - x_2) \times f(x_5)}{f(x_5) - f(x_2)} 10^{-3}$$

$$= 2.0896$$

$\therefore$ $f(x_5) = 9.1240 - 4.1792 - 5$

$= -0.0552$ (which is negative)

$f(x_2) \cdot f(x_5) < 0$

Now using equation (2) to find $x_6$

$$x_6 = x_5 - \frac{(x_5 - x_2) \times f(x_5)}{f(x_5) - f(x_2)}$$

$$= 2.0896 - \frac{(2.0896 - 3)}{(-0.0552 - 16)} \times (-0.0552)$$

$= 2.0896 + \dfrac{(0.05025)}{16.0552}$

$= 2.0927$

$\therefore \quad f(x_6) = 9.1647 - 4.1854 - 5$

$= -0.0207$ (which is negative)

So $\quad f(x_2).f(x_6) < 0$

Now using equation (2) to find $x_7$

$$x_7 = x_6 - \frac{(x_6 - x_2)}{f(x_6) - f(x_2)} \times f(x_6)$$

$$= 2.0927 - \frac{(2.0927 - 3)}{(-0.0207 - 16)} \times (-0.0207)$$

$$= 2.0927 + \frac{(-0.9073)\,(-0.0207)}{16.0207}$$

$= 2.0927 + 1.1722 \times 10^{-3}$

$= 2.0938$

Now $\quad f(x_7) = 9.1792 - 4.1876 - 5$

$= -0.0084$ (which is negative)

So $\quad f(x_2).f(x_7) < 0$

Now using equation (2) to find $x_8$

$$x_8 = x_7 - \frac{(x_7 - x_2)}{f(x_7) - f(x_2)} \times f(x_7)$$

$$= 2.0938 - \frac{(2.0938 - 3)}{(-0.0084 - 16)} \times (-0.0084)$$

$$= 2.0938 + \frac{(-0.9062)\,(-0.0084)}{16.0084}$$

$= 2.0938 + 4.755 \times 10^{-4}$

$= 2.09427$

∴ $f(x_8) = 9.1853 - 4.18854 - 5$

$= -0.00324$ (which is negative)

So $f(x_2).f(x_8) < 0$

Now using equation (2) to find $x_9$

$$x_9 = x_8 - \frac{(x_8 - x_2)}{f(x_8) - f(x_2)} \times f(x_8)$$

$$= 2.09427 - \frac{(2.09427 - 3)}{(-0.00324 - 16)} \times (-0.00324)$$

$$= 2.09427 - \frac{(-0.90573)(-0.00324)}{16.00324}$$

$$= 2.0944$$

∴ The real root of the given equation is 2.094 which is correct upto three decimals.

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# Chapter-4

# Secant Method

**Note :** In this method following formula is used to find root –

$$x_{n+1} = x_n - \frac{(x_n - x_{n-1})\, f(x_n)}{f(x_n) - f(x_{n-1})} \quad \text{\_ \_ \_ (1)}$$

**Q.1. Find the root of the equation $x^3 - 5x^2 - 17x + 20$ [use Secant Method] correct upto four decimals.**

**Ans.:** Given $f(x) = x^3 - 5x^2 - 17x + 20$ _ _ _ (2)

Taking x = 0 in equation (1)

$f(0) = 20$

Now taking x = 1

$f(1) = 1 - 5 - 17 + 20$

$= -1$

Since $f(0) = 20$ (positive) and $f(1) = -1$ (which is negative) so the root of the given equation lies between 0 and 1.

Let $x_1 = 0$ and $x_2 = 1$

$\therefore \quad f(x_1) = 20$ and $f(x_2) = -1$

using equation (1) to find $x_3$

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} \times f(x_2)$$

$$= 1 - \frac{(1 - 0)}{(-1) - 20} \times (-1)$$

$$= 1 + \frac{(1)}{(-21)} = 1 - \frac{1}{21}$$

$$= 0.9523$$

$\therefore$ $f(x_3) = f(0.9523)$

$$= (0.9523)^3 - 5\,(0.9523)^2 - 17\,(0.9523) + 20$$

$$= 0.8636 - 4.5343 - 16.1891 + 20$$

$$= 0.1402 \text{ (which is positive)}$$

Using equation (1) to find $x_4$

$$x_4 = x_3 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} \times f(x_3)$$

$$= 0.9523 - \frac{(0.9523 - 1)}{[0.1402 - (-1)]} \times 0.1402$$

$$= 0.9523 - \frac{(-0.0477)\,(0.1402)}{(1.1402)}$$

$$= 0.9523 + 0.005865 \qquad = 0.9581$$

$$f(x_4) = (0.9581)^3 - 5\,(0.9581)^2 - 17\,(0.9581) + 20$$

$$= 0.8794 - 4.5897 - 16.2877 + 20$$

$$= 0.0020 \text{ (which is positive)}$$

$$x_5 = x_4 - \frac{(x_4 - x_3)}{f(x_4) - f(x_3)} \times f(x_4)$$

$$= 0.9581 - \frac{(0.9581 - 0.9523)}{(0.0020) - (0.1402)} \times 0.0020$$

$$= 0.9581$$

Hence the root of the given equation is 0.9581 which is correct upto four decimal.

**Q.2. Given that one of the root of the non-linear equation cos x – xe$^x$ = 0 lies between 0.5 and 1.0 find the root correct upto three decimal places, by Secant Method.**

**Ans.:** Given equation is $f(x) = \cos x - xe^x$

And $x_1 = 0.5$ and $x_2 = 1.0$

$$f(x_1) = \cos(0.5) - (0.5)\,e^{0.5}$$
$$= 0.87758 - 0.82436$$
$$= 0.05321$$

Now $f(x_2) = \cos(1) - (1)\,e^1$
$$= 0.54030 - 2.71828$$
$$= -2.1780$$

Now to calculate $x_3$ using equation (1)

$$x_3 = x_2 - \frac{(x_2 - x_1)}{f(x_2) - f(x_1)} \times f(x_2)$$
$$= 1 - \frac{(1-0.5)}{(-2.1780-0.05321)} \times (-2.1780)$$
$$= 1 - \frac{(0.5)\,(2.1780)}{2.23121}$$
$$= 1 - 0.48807$$
$$= +0.51192$$

$\therefore$ $f(x_3) = f(0.51192)$
$$= \cos(0.51192) - (0.51192)e^{0.51192}$$
$$= 0.87150 - 0.85413$$
$$= 0.01767$$

Now for calculating $x_4$ using equation (1)

$$x_4 = x_3 - \frac{(x_3 - x_2)}{f(x_3) - f(x_2)} \times f(x_3)$$

$$= 0.51192 - \frac{(0.51192 - 1)}{(0.1767) - (-2.1780)} \times 0.01767$$

$$= 0.51192 - \frac{(-0.48808)\,(0.01767)}{2.19567}$$

$$= 0.51192 + \frac{0.0086243}{2.19567}$$

$$= 0.51192 + 0.003927$$

$$= 0.51584$$

$\therefore \quad f(x_4) = \cos(0.51584) - (0.51584)e^{0.51584}$

$= 0.86987 - 0.86405$

$= 0.005814$ (which is positive)

Now for calculating $x_5$ using equation (1)

$$x_5 = x_4 - \frac{(x_4 - x_3)}{f(x_4) - f(x_3)} \times f(x_4)$$

$$= 0.51584 - \frac{(0.51584 - 0.51192)}{(0.005814 - 0.01767)} \times 0.005814$$

$$= 0.51584 - \frac{0.00392}{(-0.01185)} \times (0.005814)$$

$$= 0.51584 + 0.001923$$

$$= 0.51776$$

$$= 0.5178$$

Now $f(x_5) = \cos(0.5178) - (0.5178)e^{0.5178}$

$= 0.8689 - 0.8690$

$= -0.00001$

$= -0.0000$ (upto four decimals)

Hence the root of the given equation is x = 0.5178 (which is correct upto four decimal places)

This process cannot be proceed further because $f(x_5)$ vanishes.

□ □ □

## Chapter-5

# Newton Raphson Method

**Hint :** Formula uses in this method is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

**Q.1. Find the root of the equation $x^2 - 5x + 2 = 0$ correct upto 5 decimal places. (use Newton Raphson Method.)**

**Ans.: :** Given $f(x) = x^2 - 5x + 2 = 0$

Taking x = 0

$f(0) = 2$ (which is positive)

Taking x = 1

$f(1) = 1 - 5 + 2 = -2$ (which is negative)

$f(0) . f(1) < 0$

∴ The root of the given equation lies between 0 and 1

Taking initial approximation as

$$x_1 = \frac{0+1}{2} = 0.5$$

$f(x) = x^2 - 5x + 2$

$f'(x) = 2x - 5$

Since $x_1 = 0.5$

$f(x_1) = (0.5)^2 - 5(0.5) + 2$

$= 0.25 - 2.5 + 2$

$= -0.25$

$f'(x_1) = 2(0.5) - 5$

$= 1 - 5$

$= -4$

Now finding $x_2$

$$x_2 = 0.5 - \frac{(-0.25)}{-4}$$

$$= 0.5 - \frac{0.25}{4}$$

$= 0.4375$

$f(x_2) = (0.4375)^2 - 5(0.4375) + 2$

$= 0.19140 - 2.1875 + 2$

$= 0.003906$

$f'(x_2) = 2(0.4375) - 5$

$= -4.125$

Now finding $x_3$

$$x_3 = x_2 - \frac{f(x_2)}{f^1(x_2)}$$

$$= 0.4375 - \frac{0.003906}{(-4.125)}$$

$= 0.4375 + 0.0009469$

$= 0.43844$

$f(x_3) = (0.43844)^2 - 5(0.43844) + 2$

$= 0.19222 - 2.1922 + 2$

$= 0.00002$

$f'(x_3) = 2 \times (0.43844) - 5$

$= -4.12312$

$$x_4 = x_3 - \frac{f(x_3)}{f^1(x_3)}$$

$$= 0.43844 - \frac{0.00002}{(-4.12312)}$$

$$= 0.43844 + 0.00000485$$

$$= 0.43844$$

Hence the root of the given equation is 0.43844 which is correct upto five decimal places.

**Q.2. Apply Newton Raphson Method to find the root of the equation 3x – cos x – 1 = 0 correct the result upto five decimal places.**

**Ans.:** Given equation is

$$f(x) = 3x - \cos x - 1$$

Taking x = 0

$$f(0) = 3(0) - \cos 0 - 1$$

$$= -2$$

Now taking x = 1

$$f(1) = 3(1) - \cos(1) - 1$$

$$= 3 - 0.5403 - 1$$

$$= 1.4597$$

Taking initial approximation as

$$x_1 = \frac{0+1}{2} = 0.5$$

$$f(x) = 3x - \cos x - 1$$

$$f'(x) = 3 + \sin x$$

At $x_1 = 0.5$

$$f(x_1) = 3(0.5) - \cos(0.5) - 1$$

$$= 1.5 - 0.8775 - 1$$

$= -0.37758$

$f'(x_1) = 3 - \sin(0.5)$

$= 3.47942$

Now to find $x_2$ using following formula

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.5 - \frac{(-0.37758)}{(3.47942)}$$

$= 0.5 + 0.10851$

$= 0.60852$

$f(x_2) = 3\,(0.60852) - \cos(0.60852) - 1$

$= 1.82556 - 0.820494 - 1$

$= 0.005066$

$f'(x_2) = 3 + \sin(0.60852)$

$= 3.57165$

Now finding $x_3$

$$x_3 = 0.60852 - \frac{(0.005066)}{(3.57165)}$$

$= 0.60852 - 0.0014183$

$= 0.60710$

$f(x_3) = 3\,(0.60710) - \cos(0.60710) - 1$

$= 1.8213 - 0.821305884 - 1$

$= -0.00000588$

$f'(x_3) = 3 + \sin(0.60710)$

$= 3 + 0.57048$

$= 3.5704$

Now to find $x_4$ using following formula

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$

$$= 0.60710 - \frac{(-0.00000588)}{3.5704}$$

$$= 0.60710 + 0.00000164$$

$$= 0.60710$$

Which is same as $x_3$

Hence the root of the given equation is x = 0.60710 which is correct upto five decimal places.

□ □ □

# Chapter-6

# Iterative Method

**Q.1. Find a root of the equation $x^3 + x^2 - 1 = 0$ in the interval (0,1) with an accuracy of $10^{-4}$.**

**Ans.:** Given equation is $f(x) = x^3 + x^2 - 1 = 0$

Rewriting above equation in the form

$$x = \phi(x)$$

The given equation can be expressed in either of the form :

(i) $x^3 + x^2 - 1 = 0$

$x^3 + x^2 = 1$

$x^2 (x + 1) = 1$

$$x^2 = \frac{1}{1 + x}$$

$$x = \frac{1}{\sqrt{(1+x)}} \qquad \text{--- (1)}$$

(ii) $x^3 + x^2 - 1 = 0$

$x^2 = 1 - x^3$

$$x = (1 + x^3)^{-(1/2)} \qquad \text{--- (2)}$$

(iii) $x^3 + x^2 - 1 = 0$

$x^3 = 1 - x^2$

$$x = (1 - x^2)^{1/3} \qquad \text{--- (3)}$$

Comparing equation (1) with x – g (x) = 0 we find that

$$g(x) = \frac{1}{\sqrt{(1+x)}}$$

$$g(x) = (1 + x)^{-1/2}$$

$$g'(x) = -\tfrac{1}{2}\,(1 + x)^{-3/2}$$

$$|g'(x)| = \tfrac{1}{2}\,(1 + x)^{3/2}$$

$$= \frac{1}{2(1+x)^{3/2}} < 1$$

Now comparing equation (2) with x – g(x) = 0

We find that $g(x) = (1 - x^3)^{1/2}$

$$g'(x) = \tfrac{1}{2}\,(1 + x^3)^{-1/2} \times (-3x^2)$$

$$= \frac{-3}{2}\,\frac{0+1}{2}$$

$$|g'(x)| = \frac{3}{2}\,\frac{x^2}{(1-x^2)^{1/2}}$$

Which is not less than one.

Now comparing equation (3) with x – g(x) = 0

$$g(x) = (1 - x^2)^{1/3}$$

$$g'(x) = \frac{1}{3}\,(1 - x^2)^{-2/3} \times (-2x)$$

$$= -\frac{2}{3}\,\frac{x}{(1-x^2)^{1/2}}$$

$$|g'(x)| = \frac{2}{3}\,\frac{x}{(1-x^2)^{2/3}}$$

Which is not less than one.

Hence this method is applicable only to equation (1) because it is convergent for all $x \in (0, 1)$

Now taking initial approximation

$$x_1 = \frac{0+1}{2} = 0.5$$

So $x_2 = \frac{1}{\sqrt{(1+x_1)}}$ [using iteration scheme $x_{n+1} = \frac{1}{\sqrt{(x_n+1)}}$ ]

$$x_2 = \frac{1}{\sqrt{0.5+1}} = \frac{1}{\sqrt{1.5}} = 0.81649$$

Similarly

$$x_3 = \frac{1}{\sqrt{(x_2+1)}} = \frac{1}{\sqrt{0.81649+1}} = 0.7419$$

$$x_4 = \frac{1}{\sqrt{(x_3+1)}} = \frac{1}{\sqrt{0.7419+1}} = 0.7576$$

$$x_5 = \frac{1}{\sqrt{(x_4+1)}} = \frac{1}{\sqrt{0.7576+1}} = 0.7542$$

$$x_6 = \frac{1}{\sqrt{(x_5+1)}} = \frac{1}{\sqrt{0.7542+1}} = 0.7550$$

$$x_7 = \frac{1}{\sqrt{(x_6+1)}} = \frac{1}{\sqrt{0.7550+1}} = 0.7548$$

$$x_8 = \frac{1}{\sqrt{(x_7+1)}} = \frac{1}{\sqrt{0.7548+1}} = 0.7548$$

Hence the approximate root of the given equation is x = 0.7548

**Q.2. Find the root of the equation 2x = cos x + 3 correct upto 3 decimal places.**

**Ans.:** Given equation is

$$f(x) = 2x - \cos x - 3 = 0$$

Rewriting above equation in the form x = g(x)

$$\Rightarrow 2x = \cos x + 3$$

$$\Rightarrow x = \frac{\cos x + 3}{2} \qquad \text{---} (1)$$

Comparing above equation with the following equation x = g(x) we find the

$$g(x) = \frac{\cos x + 3}{2} = \frac{\cos x}{2} + \frac{3}{2}$$

$$g'(x) = \frac{-\sin x}{2}$$

$$|g'(x)| = \frac{\sin x}{2}$$

For $x \in (1, 2)$

$$|\sin x| < 1$$

Hence the iterative scheme $x_{n+1} = \frac{\cos(x_n) + 3}{2}$ is convergent.

Now taking initial approximation $x_1 = 1.5$

$$\therefore \quad x_2 = \frac{\cos x_1 + 3}{2} = \frac{\cos(1.5) + 3}{2} = 1.5353$$

$$x_3 = \frac{\cos(x_2) + 3}{2} = \frac{\cos(1.5353) + 3}{2} = 1.5177$$

$$x_4 = \frac{\cos(x_3) + 3}{2} = \frac{\cos(1.5177) + 3}{2} = 1.5265$$

$$x_5 = \frac{\cos(x_4) + 3}{2} = \frac{\cos(1.5265) + 3}{2} = 1.5221$$

$$x_6 = \frac{\cos(x_5) + 3}{2} = \frac{\cos(1.5221) + 3}{2} = 1.5243$$

$$x_7 = \frac{\cos(x_6) + 3}{2} = \frac{\cos(1.5243) + 3}{2} = 1.5230$$

$$x_8 = \frac{\cos(x_7) + 3}{2} = \frac{\cos(1.5230) + 3}{2} = 1.523$$

Which is same as $x_7$

Hence the root of the given equation is x = 1.523 (which is correct upto 3 decimals)

**Q.3. Find the root of the equation $xe^x = 1$ in the internal (0, 1) (use iterative Method)**

**Ans.:** Given equation is $xe^x - 1 = 0$

Rewriting above equation in the form of x = g (x)

$xe^x - 1 = 0$

$xe^x = 1$

$x = e^{-x}$

Comparing it with the equation x = g (x) we find that

$g(x) = e^{-x}$

$g'(x) = -e^{-x}$

$|g'(x)| = e^{-x} < 1$

Hence the iterative scheme is

$x_{n+1} = e^{-x_n}$

Now taking initial approximation

$x_1 = 0.5$

$x_2 = e^{-x_1} = e^{-(0.5)} = 0.60653$

$x_3 = e^{-x_2} = e^{-(0.6065)} = 0.5452$

$x_4 = e^{-x_3} = e^{-(0.5452)} = 0.5797$

$x_5 = e^{-x_4} = e^{-0.5797} = 0.5600$

$x_6 = e^{-x_5} = e^{-0.5600} = 0.5712$

$x_7 = e^{-x_6} = e^{-(0.5712)} = 0.5648$

$x_8 = e^{-x_7} = e^{-(0.5648)} = 0.5684$

$x_9 = e^{-x_8} = e^{-(0.5684)} = 0.5664$

$x_{10} = e^{-x_9} = e^{-(0.5664)} = 0.5675$

Now

$x_{11} = e^{-x_{10}} = e^{-0.5675} = 0.5669$

$x_{12} = e^{-x_{11}} = e^{-0.5669} = 0.5672$

$x_{13} = e^{-x_{12}} = e^{-(0.5672)} = 0.5671$

$x_{14} = e^{-x_{13}} = e^{-(0.5671)} = 0.5671$

Hence the approximate root the given equation is x = 0.5671

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# Chapter-7

# Gauss Elimination Method

**Q.1. Use gauss elimination method to solve :**

**$x + y + z = 7$**

**$3x + 3y + 4z = 24$**

**$2x + y + 3z = 16$**

**Ans.:** Since in the first column the largest element is 3 in the second equation, so interchanging the first equation with second equation and making 3 as first pivot.

$3x + 3y + 4z = 24$ _ _ _ (1)

$x + y + z = 7$ _ _ _ (2)

$2x + y + 3z = 16$ _ _ _ (3)

Now eliminating x form equation (2) and equation (3) using equation (1)

-3 × equation (2) + 2 × equation (1), 3 × equation (3) – 2 × equation (1)

we get

$$\begin{array}{r} -3x - 3y - 3z = -21 \\ 3x + 3y + 4z = 24 \\ \hline z = 3 \end{array} \quad \text{and} \quad \begin{array}{r} 6x + 3y + 9z = 48 \\ -6x \pm 6y \pm 8z = 48 \\ \hline -3y + z = 0 \end{array}$$

$= 3y - z = 0$

$3x + 3y + 4z = 24$ _ _ _ (4)

$z = 3$ _ _ _ (5)

$3y – z = 0$ _ _ _ (6)

Now since the second row cannot be used as the pivot row since $a_{22} = 0$ so interchanging the equation (5) and (6) we get

$3x + 3y + 4z = 24$ ___ (7)

$3y - z = 0$ ___ (8)

$z = 3$ ___ (9)

Now it is upper triangular matrix system. So by back substitution we obtain.

$\boxed{z = 3}$

From equation (8)

$3y - 3 = 0$

$3y = 3$

$\boxed{y = 1}$

From equation (7)

$3x + 3(1) + 4(3) = 24$

$3x + 3 + 12 = 24$

$3x + 15 = 24$

$3x = 9$

$\boxed{x = 3}$

Hence the solution fo given system of linear equation is

$x = 3$ , $y = 1$ , $z = 3$

**Q.2. Solve the following system of linear equation by Gauss Elimination Method :**

$\mathbf{2x_1 + 4x_2 + x_3 = 3}$

$\mathbf{3x_1 + 2x_2 - 2x_3 = -2}$

$\mathbf{x_1 - x_2 + x_3 = 6}$

**Ans.:** Since in the first column the largest element is 3 in the second row, so interchanging first equation with second equation and making 3 as first pivot.

$3x_1 + 2x_2 - 2x_3 = -2$ ___ (1)

$2x_1 + 4x_2 + x_3 = 3$ ——— (2)

$x_1 - x_2 + x_3 = 6$ ——— (3)

Eliminating $x_1$ form equation (2) and equation (3) using equation (1)

-3 × equation (2) + 2 × equation (1) and + 3 × equation (3) – equation (1)

$$\begin{array}{r} -6x_1 - 12x_2 - 3x_3 = -9 \\ 6x_1 + 4x_2 - 4x_3 = -4 \\ \hline -8x_2 - 7x_3 = -13 \end{array} \quad \text{and} \quad \begin{array}{r} 3x_1 - 3x_2 + 3x_3 = 18 \\ 3x_1 + 2x_2 - 2x_3 = -2 \\ - \quad - \quad + \quad + \\ \hline -5x_2 + 5x_3 = 20 \end{array} \quad +$$

$8x_2 + 7x_3 = 13$ $x_2 - x_3 = -4$

So the system now becomes :

$3x_1 + 2x_2 - 2x_3 = -2$ ——— (4)

$8x_2 + 7x_3 = 13$ ——— (5)

$x_2 - x_3 = -4$ ——— (6)

Now eliminating $x_2$ from equation (6) using equation (5) {8 × equation (6) – equation (5)}

$$\begin{array}{r} 8x_2 - 8x_3 = -32 \\ - \quad 8x_2 + 7x_3 = -13 \\ - \quad - \quad + \\ \hline -15x_3 = -45 \end{array}$$

$x_3 = 3$

So the system of linear equation is

$3x_1 + 2x_2 - 2x_3 = -2$ ——— (7)

$8x_2 + 7x_3 = 13$ ——— (8)

$x_3 = 3$ ——— (6)

Now it is upper triangular system so by back substitution we obtain

$x_3 = 3$

From equation (8)

$8x_2 + 7(3) = 13$

$8x_2 = 13 - 21$

$8x_2 = -8$

$x_2 = -1$

From equation (9)

$3x_1 + 2(-1) - 2(3) = -2$

$3x_1 = -2 + 2 + 6$

$3x_1 = 6$

$x_1 = 2$

$\therefore$ Hence the solution of the given system of linear equation is :

$x_1 = 2$ , $x_2 = -1$ , $x_3 = 3$

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# Chapter-8

# Gauss-Jordan Elimination Method

**Q.1. Solve the following system of equations :**

$$10x_1 + 2x_2 + x_3 = 9 \qquad \_\_\_ (1)$$

$$2x_1 + 20x_2 - 2x_3 = -44 \qquad \_\_\_ (2)$$

$$-2x_1 + 3x_2 + 10x_3 = 22 \qquad \_\_\_ (3)$$

**Use Gauss Jordan Method.**

**Ans.:** Since in the given system pivoting is not necessary. Eliminating $x_1$ from equation (2) and equation (3) using equation (1)

5 × equation (2) – equation (1) , 5 × equation (3) + equation (1)

$$\begin{array}{r} 10x_1 - 100x_2 - 10x_3 = -220 \\ \_\ 10x_1 \ +\ 2x_2 \ +\ x_3 = \_\ 9 \\ \hline 98x_2 - 11x_3 = -229 \end{array} \quad \text{and} \quad \begin{array}{r} -10x_1 + 15x_2 + 50x_3 = 110 \\ 10x_1 + 2x_2 + x_3 = 9 \\ \hline 17x_2 + 51x_3 = 119 \\ = x_2 + 3x_3 = 7 \end{array}$$

Now the system of equation becomes

$$10x_1 + 2x_2 + x_3 = 9 \qquad \_\_\_ (4)$$

$$98x_2 - 11x_3 = -229 \qquad \_\_\_ (5)$$

$$x_2 + 3x_3 = 7 \qquad \_\_\_ (6)$$

Now eliminating $x_2$ from equation (4) and (6) using equation (5)

98 × equation (6) – equation (5) , 49 × equation (4) - equation (5)

$$\begin{array}{rl} 98x_2 + 294x_3 &= 686 \\ -98x_2 \mp 11x_3 &= \mp 229 \\ \hline 305x_3 &= 915 \end{array}$$

$$x_3 = 3$$

$$\begin{array}{rl} 490x_1 + 98x_2 + 49x_3 &= 441 \\ -\ 98x_2 - 11x_3 &= 9 \\ \hline 490x_1 + 60x_3 &= 670 \end{array}$$

$$= 49x_1 + 6x_3 = 67$$

Now the system of equation becomes :

$49x_1 + 0 + 6x_3 = 67$ _ _ _ (7)

$98x_2 - 11x_3 = -229$ _ _ _ (8)

$x_3 = 3$ _ _ _ (9)

Hence it reduces to upper triangular system now by back substitution.

$x_3 = 3$

From equation (8)

$98x_2 - 11 \times 3 = -229$

$98x_2 = -229 + 33$

$98x_2 = -196$

$x_2 = -2$

From equation (7)

$49x_1 + 6(3) = 67$

$49x_1 = 67 - 18$

$49x_1 = 49$

$x_1 = 1$

Thus the solution of the given system of linear equation is

$x_1 = 1$ , $x_2 = -2$ , $x_3 = 3$

**Q.2. Solve the following system of equation using Gauss-Jordan Elimination Method.**

**$2x_1 - 2x_2 + 5x_3 = 13$ _ _ _ (1)**

**$2x_1 + 3x_2 + 4x_3 = 20$ _ _ _ (2)**

**$3x_1 - x_2 + 3x_3 = 10$ _ _ _ (3)**

**Ans.:** Solve this question like question no. 17.

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# Chapter-9

# Matrix Inversion Method

**Q.1. Solve the given system of equation using Matrix inversion Method.**

$$6x_1 + 3x_2 + 7x_3 = 7$$

$$x_1 + 5x_2 + 2x_3 = -7$$

$$7x_1 + 2x_2 + 10x_3 = 13$$

**Ans.:** The given system of equations cab be written in the form of AX = B

$$A = \begin{bmatrix} 6 & 3 & 7 \\ 1 & 5 & 2 \\ 7 & 2 & 10 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ -7 \\ 13 \end{bmatrix}$$

The solution can be given by $X = A^{-1}B$ so to find the solution first we have to find $A^{-1}$ using Gauss-Jordan Method. The inverse of matrix A that is $A^{-1}$ is obtained by reducing the argumented matrix [A/I] into the matrix $[I/A^{-1}]$

The argumented matrix is given by

$$\left[\begin{matrix} 6 & 3 & 7 \\ 1 & 5 & 2 \\ 7 & 2 & 10 \end{matrix} \middle/ \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right]$$

$R_1 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 7 & 2 & 10 & 0 & 0 & 1 \\ 1 & 5 & 2 & 0 & 1 & 0 \\ 6 & 3 & 7 & 1 & 0 & 0 \end{array}\right]$$

$R_2 \rightarrow 7R_2 - R_1$ , $R_3 \rightarrow \frac{7}{6} R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 7 & 2 & 10 & 0 & 0 & 1 \\ 0 & 33 & 4 & 0 & 7 & -1 \\ 0 & \frac{3}{2} & -\frac{11}{6} & \frac{7}{6} & 0 & -1 \end{array}\right]$$

$R_1 \rightarrow -\frac{33}{2} R_1 + R_2$ , $R_3 \rightarrow \frac{-2 \text{ x } 33}{3} R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} \frac{-231}{2} & 2 & -166 & 0 & 7 & -\frac{35}{2} \\ 0 & 33 & 4 & 0 & 7 & -1 \\ 0 & 0 & \frac{133}{3} & \frac{-77}{3} & 0 & 21 \end{array}\right]$$

$R_1 \rightarrow \frac{133 R_1}{2 \text{ x } 161} + R_3$ , $R_2 \rightarrow \frac{-133}{3 \text{ x } 4} R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} \frac{-10241}{322} & 0 & 0 & \frac{-77}{3} & \frac{616}{69} & \frac{2233}{138} \\ 0 & \frac{-1463}{4} & 0 & \frac{-77}{3} & \frac{-847}{12} & \frac{385}{12} \\ 0 & 0 & \frac{133}{3} & \frac{-77}{3} & 7 & 21 \end{array}\right]$$

$R_1 \leftrightarrow \frac{-322}{-10241} R_1$ , $R_2 \leftrightarrow \frac{4}{-1463} R_2$ , $R_3 \leftrightarrow \frac{3}{133} R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{46}{57} & \frac{-16}{57} & \frac{-29}{57} \\ 0 & 1 & 0 & \frac{4}{57} & \frac{11}{57} & \frac{-5}{57} \\ 0 & 0 & 1 & \frac{-11}{19} & \frac{3}{19} & \frac{9}{19} \end{array}\right]$$

$$R_1 \leftrightarrow \frac{-322}{-10241} R_1 \quad , \quad R_2 \leftrightarrow \frac{4}{-1463} R_2 \quad , \quad R_3 \leftrightarrow \frac{3}{133} R_3$$

Hence

$$A^{-1} = \frac{1}{57}\begin{bmatrix} 46 & -16 & -29 \\ 4 & 11 & -5 \\ -33 & 9 & 27 \end{bmatrix}$$

Thus the matrix A is reduced to identity matrix Hence the solution of the given system of equations is

$$X = A^{-1} B$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{57}\begin{bmatrix} 46 & -16 & -29 \\ 4 & 11 & -5 \\ -33 & 9 & 27 \end{bmatrix}\begin{bmatrix} 7 \\ -7 \\ 13 \end{bmatrix}$$

$$= \frac{1}{57}\begin{bmatrix} 322 + 112 - 377 \\ 28 - 77 - 65 \\ 231 - 63 + 351 \end{bmatrix} = \frac{1}{57}\begin{bmatrix} 57 \\ -114 \\ 57 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

**Q.2. Solve the following system of linear equations using matrix inversion method.**

**$3x_1 + 2x_2 + 4x_3 = 7$**

**$2x_1 + x_2 + x_3 = 7$**

**$x_1 + 3x_2 + 4x_3 = 2$**

**Ans.:** The given system of linear equations can be written in the form of AX = B

$$= \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$$

The solution can be given by $X = A^{-1} B$. for this we have to first find the value of $A^{-1}$ using Gauss Jordan Method.

The inverse of the matrix A using Gauss Jordan method is obtained by reducing the argumented matrix [A/I] in the form of[I/$A^{-1}$].

The argumented matrix is given as follows :

$$\left[\begin{matrix} 3 & 2 & 4 \\ 2 & 1 & 1 \\ 1 & 3 & 4 \end{matrix} \middle/ \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right]$$

Here pivoting is not necessary.

$R_2 \to \frac{3}{2} R_2 - R_1$ , $R_3 = 3R_3 - R_1$

$$= \left[\begin{matrix} 3 & 2 & 4 \\ 0 & \frac{-1}{2} & \frac{-5}{2} \\ 0 & 7 & 8 \end{matrix} \middle/ \begin{matrix} 1 & 0 & 0 \\ -1 & \frac{3}{2} & 0 \\ -1 & 0 & 3 \end{matrix}\right]$$

$R_1 \to \frac{1}{4} R_1 + R_2$ , $R_3 = \frac{1}{14} R_3 + R_2$

$$= \left[\begin{matrix} \frac{3}{4} & 0 & \frac{-3}{2} \\ 0 & \frac{-1}{2} & \frac{-5}{2} \\ 0 & 0 & \frac{-27}{14} \end{matrix} \middle/ \begin{matrix} \frac{-3}{4} & \frac{3}{2} & 0 \\ -1 & \frac{3}{2} & 0 \\ \frac{-15}{14} & \frac{3}{2} & \frac{3}{14} \end{matrix}\right]$$

$R_1 \to \frac{-9}{7} R_1 + R_3$ , $R_2 \to \frac{-27}{35} R_2 + R_3$

$$= \left[\begin{array}{ccc|ccc} \frac{-27}{28} & 0 & 0 & \frac{-3}{28} & \frac{-3}{7} & \frac{3}{14} \\ 0 & \frac{27}{70} & 0 & \frac{-3}{10} & \frac{12}{35} & \frac{3}{14} \\ 0 & 0 & \frac{-27}{14} & \frac{-15}{14} & \frac{3}{2} & \frac{3}{14} \end{array}\right]$$

Now $R_1 \to \frac{-28}{27} R_1$ , $R_2 \to \frac{70}{27} R_2$ , $R_3 \to \frac{-14}{27} R_3$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{9} & \frac{4}{9} & \frac{-2}{9} \\ 0 & 1 & 0 & \frac{-7}{9} & \frac{8}{9} & \frac{5}{9} \\ 0 & 0 & 1 & \frac{5}{9} & \frac{-7}{9} & \frac{-1}{9} \end{array}\right]$$

Hence

$$A^{-1} = \frac{1}{9}\begin{bmatrix} 1 & 4 & -2 \\ -7 & 8 & 5 \\ 5 & -7 & -1 \end{bmatrix}$$

Thus the solution of given matrix is given by

$$X = A^{-1} B$$

i.e. $$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 1 & 4 & -2 \\ -7 & 8 & 5 \\ 5 & -7 & -1 \end{bmatrix}\begin{bmatrix} 7 \\ 7 \\ 2 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 7 & +28 & -4 \\ -49 & +56 & +10 \\ 35 & -49 & -2 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 31 \\ 17 \\ -16 \end{bmatrix}$$

Hence $x_1 = \frac{31}{9}$ , $x_2 = \frac{17}{9}$ , $x_3 = \frac{-16}{9}$

□ □ □

# Chapter-10

# Matrix Factorization Method

**Q.1. Solve the following system of linear equation using Matrix Factorization Method.**

**$3x_1 + 5x_2 + 2x_3 = 8$**

**$8x_2 + 2x_3 = -7$**

**$6x_1 + 2x_2 + 8x_3 = 26$**

**Ans.:** Above system of equation can be written in the form AX = B where

$$A = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 8 & 2 \\ 6 & 2 & 8 \end{bmatrix}; \quad B = \begin{bmatrix} 8 \\ -7 \\ 26 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Let us assume that A = LU

$$\text{Where } L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\therefore \quad LU = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12} + U_{22} & l_{21}U_{13} + U_{23} \\ l_{31}U_{11} & l_{31}U_{12} + l_{32}U_{22} & l_{31}U_{13} + l_{32}U_{23} + U_{33} \end{bmatrix}$$

Since A = LU so comparing both matrices.

$U_{11} = 3$ _ _ _ (1)

$U_{12} = 5$ _ _ _ (2)

$U_{13} = 2$ _ _ _ (3)

$l_{21}U_{11} = 0$ _ _ _ (4)

$l_{21}U_{12} + U_{22} = 8$ _ _ _ (5)

$l_{21}U_{13} + U_{23} = 2$ _ _ _ (6)

$l_{31}U_{11} = 6$ _ _ _ (7)

$l_{31}U_{12} + l_{32}U_{22} = 2$ _ _ _ (8)

$l_{31}U_{13} + l_{32}U_{22} + U_{33} = 8$ _ _ _ (9)

$l_{21}U_{11} = 0$

$\Rightarrow l_{21} \times = 0$

$\Rightarrow l_{21} = 0$ _ _ _ (10)

$l_{31}U_{11} = 6$

$\Rightarrow l_{31} \times 3 = 6$

$\Rightarrow l_{31} = 6/3$

$\Rightarrow l_{31} = 2$ _ _ _ (11)

Now from equation (5)

$l_{21}U_{12} + U_{22} = 8$

$\Rightarrow 0 \times U_{12} + U_{22} = 8$

$\Rightarrow U_{22} = 8$ _ _ _ (12)

From equation (6)

$l_{21}U_{13} + U_{23} = 2$

$\Rightarrow 0 \times U_{13} + U_{23} = 2$

$\Rightarrow U_{23} = 2$ _ _ _ (13)

From equation (8)

$l_{31}U_{12} + \Box_{32}U_{22} = 2$

$\Rightarrow 2 \times 5 + L_{32} \times 8 = 2$

$\Rightarrow l_{32} \times 8 = 2 - 10 = -8$

$\Rightarrow l_{32} = -1$

From equation (9)

$l_{31}U_{13} + l_{32}U_{23} + U_{33} = 8$

$\Rightarrow 2 \times 2 + (-1) \times 2 + U_{33} = 2$

$\Rightarrow U_{33} = 8 - 4 + 2$

$\Rightarrow U_{33} = 6$

$$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 3 & 5 & 2 \\ 0 & 8 & 2 \\ 0 & 0 & 6 \end{bmatrix}$$

Since the given system of equation can be written as AX = B [Here A = LU]

$\therefore \quad LUX = B$ ___ (14)

Now let UX = Y ___ (15)

$\therefore \quad LY = B$ ___ (16)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 26 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ y_2 \\ 2y_1 - y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 26 \end{bmatrix}$$

On comparing both matrices, we get

$y_1 = 8$ , $y_2 = -7$

and $2y_1 - y_2 + y_3 = 26$

$\Rightarrow 2 \times 8 + 7 + y_3 = 26$

$\Rightarrow y_3 = 26 - 16 - 7$

$\Rightarrow y_3 = 3$

$$\therefore \quad Y = \begin{bmatrix} 8 \\ -7 \\ 3 \end{bmatrix}$$

From equation (15)

UX =Y

$$\Rightarrow \begin{bmatrix} 3 & 5 & 2 \\ 0 & 8 & 2 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x_1 & + & 5x_2 & + & 2x_3 \\ 0 & + & 8x_2 & + & 2x_3 \\ 0 & + & 0 & + & 6x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -7 \\ 3 \end{bmatrix}$$

Comparing both matrices

$3x_1 + 5x_2 + 2x_3 = 8$ _ _ _ (17)

$8x_2 + 2x_3 = -7$ _ _ _ (18)

$6\, x_3 = 3$ _ _ _ (19)

From equation (19)

$6x_3 = 3$

$\Rightarrow x_3 = \frac{1}{2}$

From equation (18)

$8x_2 + 2x_3 = -7$

$\Rightarrow 8x_2 + 2 \times \frac{1}{2} = -7$

$\Rightarrow 8x_2 = -7 - 1$

$\Rightarrow 8x_2 = -8$

$\Rightarrow x_2 = -1$

From equation (17)

$$3x_1 + 5x_2 + 2x_3 = 8$$

$$\Rightarrow 3x_1 = 8 + 5 - 1$$

$$\Rightarrow 3x_1 = 12$$

$$\Rightarrow x_1 = 4$$

Thus the solution of given system of equation is

$$x_1 = 4 \quad , \quad x_2 = -1 \quad \text{and} \quad x_3 = \frac{1}{2}$$

**Q. 2. Solve the following system of linear equation using Matrix Factorization Method.**

$$\mathbf{x + 2y + 3z = 14}$$

$$\mathbf{3x + y + 2z = 11}$$

$$\mathbf{2x + 3y + z = 11}$$

**Ans.:** Above system of equation can be written in the form of AX = B where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \quad ; \quad B = \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Let us assume that A = LU where

$$L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

$$\Rightarrow LU = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ l_{21}U_{11} & l_{21}U_{12} + U_{22} & l_{21}U_{13} + U_{23} \\ l_{31}U_{11} & l_{31}U_{12} + l_{32}U_{22} & l_{31}U_{13} + l_{32}U_{23} + U_{33} \end{bmatrix}$$

Since A = LU so comparing both sides we get

$U_{11} = 1$ _ _ _ (1)

$U_{12} = 2$ _ _ _ (2)

$U_{13} = 3$ _ _ _ (3)

$l_{21}U_{11} = 3$ _ _ _ (4)

$l_{21}U_{12} + U_{22} = 1$ _ _ _ (5)

$l_{21}U_{13} + U_{23} = 2$ _ _ _ (6)

$l_{31}U_{11} = 2$ _ _ _ (7)

$l_{31}U_{12} + l_{32}U_{22} = 3$ _ _ _ (8)

$l_{31}U_{13} + l_{32}U_{23} + U_{33} = 1$ _ _ _ (9)

From equation (3)

$l_{21}U_{11} = 3$

$\Rightarrow l_{21} \times 1 = 3$

$\Rightarrow l_{21} = 3$ _ _ _ (10)

From equation (5)

$l_{21}U_{12} + U_{22} = 1$

$\Rightarrow 3 \times 2 + U_{22} = 1$

$\Rightarrow U_{22} = 1 - 6$

$\Rightarrow U_{22} = -5$ _ _ _ (11)

From equation (6)

$l_{21}U_{13} + U_{23} = 2$

$\Rightarrow 3 \times 3 + U_{23} = 2$

$\Rightarrow U_{23} = 2 - 9$

$\Rightarrow U_{23} = -7$ _ _ _ (12)

From equation (7)

$l_{31}U_{11} = 2$

$\Rightarrow l_{31} \times 1 = 2$

$\Rightarrow l_{31} = 2$ _ _ _ (13)

From equation (8)

$l_{31}U_{12} + l_{32}U_{22} = 3$

$\Rightarrow 2\times 2 + l_{32}(-5) = 3$

$\Rightarrow l_{32}\times(-5) = 3-4$

$\Rightarrow l_{32} = \dfrac{1}{5}$ ——— (14)

From equation (9)

$l_{31}U_{13} + l_{32}U_{23} + U_{33} = 1$

$\Rightarrow 2\times 3 + \dfrac{1}{5}\times(-7) + U_{33} = 1$

$\Rightarrow 6 - \dfrac{7}{5} + U_{33} = 1$

$\Rightarrow U_{33} = 1 - 6 + \dfrac{7}{5} = -5 + \dfrac{7}{5} = \dfrac{-25+7}{5}$

$\Rightarrow U_{33} = \dfrac{-18}{5}$ ——— (15)

$$\therefore \quad L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{1}{5} & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & 0 & \frac{-18}{5} \end{bmatrix}$$

We know that AX = B

$\Rightarrow LUX = B$ where [A = LU] ——— (16)

Now let UX = Y ——— (17)

So LY = B ——— (18)

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \frac{1}{5} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ 3y_1 + y_2 \\ 2y_1 + \frac{1}{5}y_2 + y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix}$$

Comparing both sides we get

$$y_1 = 14 \quad \text{---} (19)$$

$$3y_1 + y_2 = 11 \quad \text{---} (20)$$

$$2y_1 + \frac{1}{5} y_2 + y_3 = 11 \quad \text{---} (21)$$

From equation (20)

$$3y_1 + y_2 = 11$$

$$\Rightarrow 3 \times 14 + y_2 = 11$$

$$\Rightarrow y_2 = 11 - 42$$

$$\Rightarrow y_2 = -31$$

From equation (21)

$$\Rightarrow 2(14) + \frac{1}{5}(-31) + y_3 = 11$$

$$\Rightarrow 28 - \frac{31}{5} + y_3 = 11$$

$$\Rightarrow y_3 = 11 - 28 + \frac{31}{5}$$

$$\Rightarrow y_3 = -17 + \frac{31}{5}$$

$$\Rightarrow y_3 = \frac{-85 + 31}{5}$$

$$\Rightarrow y_3 = \frac{-54}{5}$$

$$\therefore \quad Y = \begin{bmatrix} 14 \\ -31 \\ \frac{-54}{5} \end{bmatrix}$$

Since UX = Y

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \\ 0 & 0 & \frac{-18}{5} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -31 \\ \frac{-54}{5} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x + 2y + 3z \\ 0 - 5y + 7z \\ 0 + 0 - \frac{18}{5}z \end{bmatrix} = \begin{bmatrix} 14 \\ -31 \\ \frac{-54}{5} \end{bmatrix}$$

$$x + 2y + 3z = 14 \qquad --- (22)$$

$$-5y - 7z = -31 \qquad --- (23)$$

$$\frac{-18}{5} z = \frac{-54}{5} \qquad --- (24)$$

$$\Rightarrow z = \frac{54}{5} \times \frac{5}{18}$$

$$\Rightarrow z = 3$$

From equation (23)

$$5y + 7z = 31$$

$$\Rightarrow 5y + 7 \times 3 = 31$$

$$\Rightarrow 5y = 31 - 21 = 10$$

$$\Rightarrow y = 2$$

From equation (22)

$$x + 2y + 3z = 14$$

$$\Rightarrow x + 4 + 9 = 14$$

$$\Rightarrow x = 14 - 13$$

$$\Rightarrow x = 1$$

Thus the solution of the given system of equation is

$x = 1$ ; $y = 2$ and $z = 3$

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# Chapter-11

# Jacobi Method

**Q.1. Solve the following system of equation by Jacobi Method.**

$$83x_1 + 11x_2 - 4x_3 = 95$$

$$7x_1 + 52x_2 + 13x_3 = 104$$

$$3x_1 + 8x_2 + 29x_3 = 71$$

**Ans.:** Since the given system of equation is

$$83x_1 + 11x_2 - 4x_3 = 95 \quad \text{\_ \_ \_ (1)}$$

$$7x_1 + 52x_2 + 13x_3 = 104 \quad \text{\_ \_ \_ (2)}$$

$$3x_1 + 8x_2 + 29x_3 = 71 \quad \text{\_ \_ \_ (3)}$$

The diagonal elements in the given system of linear equations is not zero so the equation (1), (2) and (3) can be written as :

$$x_1^{(n+1)} = \frac{1}{83}[95 - 11x_2^{(n)} + 4x_3^{(n)}]$$

$$x_2^{(n+1)} = \frac{1}{52}[104 - 7x_1^{(n)} - 13x_3^{(n)}] \text{ and}$$

$$x_3^{(n+1)} = \frac{1}{29}[71 - 3x_1^{(n)} - 8x_2^{(n)}]$$

Now taking initial approximation as :

$$x_1^{(0)} = 0 \quad ; \quad x_2^{(0)} = 0 \quad \text{and} \quad x_3^{(0)} = 0$$

Now for first approximation :

$$x_1^{(1)} = \frac{1}{83}[95 - 11x_2^{(0)} + 4x_3^{(0)}] = 1.1446$$

$$x_2^{(1)} = \frac{1}{52}[104 - 7x_1^{(0)} - 13x_3^{(0)}] = 2$$

$$x_3^{(1)} = \frac{1}{29}[71 - 3x_1^{(0)} - 8x_2^{(0)}] = 2.4483$$

Similarly second approximation :

$$x_1^{(2)} = \frac{1}{83}[95 - 11x_2^{(1)} + 4x_3^{(1)}]$$

$$= \frac{1}{83}[95 - 11(2) + 4(2.4483)] = 0.9975$$

$$x_2^{(2)} = \frac{1}{52}[104 - 7x_1^{(1)} - 13x_3^{(1)}]$$

$$= \frac{1}{52}[104 - 7(1.1446) - 13(2.4483)] = 1.2338$$

$$x_3^{(2)} = \frac{1}{29}[71 - 3x_1^{(1)} - 8x_2^{(1)}]$$

$$= \frac{1}{29}[71 - 3(1.1446) - 8 \times 2] = 1.7781$$

Now the third iteration :

$$x_1^{(3)} = \frac{1}{83}[95 - 11x_2^{(2)} + 4x_3^{(2)}]$$

$$= \frac{1}{83}[95 - 11\times(1.2338) + 4(1.7781)] = 1.0668$$

$$x_2^{(3)} = \frac{1}{52}[104 - 7x_1^{(2)} - 13x_3^{(2)}]$$

$$= \frac{1}{52}[104 - 7\times(0.9975) - 13\times(1.7781)] = \frac{1}{52}[73.9022]$$

$$= 1.4212$$

$$x_3^{(3)} = \frac{1}{29}[71 - 3x_1^{(2)} - 8x_2^{(2)}]$$

$$= \frac{1}{29}[71 - 3 \times (0.9975) - 8 \times (1.2338)] = 2.0047$$

Similarly other iterations are :

$x_1^{(4)} = 1.0528$

$x_2^{(4)} = 1.3552$

$x_3^{(4)} = 1.9459$

$x_1^{(5)} = 1.0588$

$x_2^{(5)} = 1.3718$

$x_3^{(5)} = 1.9655$

$x_1^{(6)} = 1.0575$

$x_2^{(6)} = 1.3661$

$x_3^{(6)} = 1.9603$

$x_1^{(7)} = 1.0580$

$x_2^{(7)} = 1.3676$

$x_3^{(7)} = 1.9620$

$x_1^{(8)} = 1.0579$

$x_2^{(8)} = 1.3671$

$x_3^{(8)} = 1.9616$

$x_1^{(9)} = 1.0579$

$x_2^{(9)} = 1.3671$

$x_3^{(9)} = 1.9616$

Thus the values obtained by successive iteration is given by following table :

| x | $x_1^{(n)}$ | $x_2^{(n)}$ | $x_3^{(n)}$ | $x_1^{(n+1)}$ | $x_2^{(n+1)}$ | $x_3^{(n+1)}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1.1446 | 2 | 2.4483 |
| 1 | 1.1446 | 2 | 2.4483 | 0.9975 | 1.2338 | 1.7781 |
| 2 | 0.9975 | 1.2338 | 1.7781 | 1.0667 | 1.4211 | 2.0047 |
| 3 | 1.0667 | 1.4211 | 2.0047 | 1.0528 | 1.3552 | 1.9459 |
| 4 | 1.0528 | 1.3552 | 1.9459 | 1.0587 | 1.3718 | 1.9655 |
| 5 | 1.0587 | 1.3718 | 1.9655 | 1.0575 | 1.3661 | 1.9603 |
| 6 | 1.0575 | 1.3661 | 1.9603 | 1.0580 | 1.3676 | 1.9620 |
| 7 | 1.0580 | 1.3676 | 1.9620 | 1.0579 | 1.3671 | 1.9616 |
| 8 | 1.0579 | 1.3671 | 1.9616 | 1.0579 | 1.3671 | 1.9616 |

Thus the solution is

$x_1 = 1.0579$ ; $x_2 = 1.3671$ and $x_3 = 1.9616$

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# Chapter-12

# Gauss Seidel Method

**[This method is also called the method of successive displacement]**

**Q.1. Solve the following linear equation :**

$$2x_1 - x_2 + x_3 = 5$$

$$x_1 + 2x_2 + 3x_3 = 10$$

$$x_1 + 3x_2 - 2x_3 = 7$$

**(Use Gauss Seidel Method)**

**Ans.:** Above system of equations can be written as :

$$2x_1 - x_2 + x_3 = 5 \quad \text{\_ \_ \_ (1)}$$

$$x_1 + 3x_2 - 2x_3 = 7 \quad \text{\_ \_ \_ (2)}$$

$$x_1 + 2x_2 + 3x_3 = 10 \quad \text{\_ \_ \_ (3)}$$

Iterative equations are :

$$x_1^{(n+1)} = \frac{1}{2}[5 + x_2^{(n)} - x_3^{(n)}] \quad \text{\_ \_ \_ (4)}$$

$$x_2^{(n+1)} = \frac{1}{3}[7 - x_1^{(n+1)} + 2x_3^{(n)}] \quad \text{\_ \_ \_ (5)}$$

$$x_3^{(n+1)} = \frac{1}{3}[10 - x_1^{(n+1)} - 2x_2^{(n+1)}] \quad \text{\_ \_ \_ (6)}$$

Taking initial approximations as :

$x_1^{(0)} = 0$ ; $x_2^{(0)} = 0$ and $x_3^{(0)} = 0$

First approximation is :

$$x_1^{(1)} = \frac{1}{2}[5 + x_2^{(0)} - x_3^{(0)}]$$

$$= \frac{1}{2}[5 + 0 - 0] = \frac{5}{2} = 2.5$$

$$x_2^{(1)} = \frac{1}{3}[7 - x_1^{(1)} + 2x_3^{(0)}]$$

$$= \frac{1}{3}[7 - 2.5 + 2 \times 0] = \frac{1}{3}(4.5) = 1.5$$

$$x_3^{(1)} = \frac{1}{3}[10 - x_1^{(1)} - 2x_2^{(1)}]$$

$$= \frac{1}{3}[10 - 2.5 - 2 \times 1.5] = 1.5$$

Now second approximation :

$$x_1^{(2)} = \frac{1}{2}[5 + x_2^{(1)} - x_3^{(1)}]$$

$$= \frac{1}{2}[5 + (1.5) - 1.5] = 2.5$$

$$x_2^{(2)} = \frac{1}{3}[7 - x_1^{(2)} + 2x_3^{(1)}]$$

$$= \frac{1}{3}[7 - 2.5 + 2(1.5)] = 2.5$$

$$x_3^{(2)} = \frac{1}{3}[10 - x_1^{(2)} - 2x_2^{(2)}]$$

$$= \frac{1}{3}[10 - 2.5 - 2 \times 2.5] = 0.8333$$

$$x_1^{(3)} = \frac{1}{2}[5 + x_2^{(2)} - x_3^{(2)}]$$

$$= \frac{1}{2}[5 + 2.5 - 0.8333] = 3.3333$$

$$x_2^{(3)} = \frac{1}{3}[7 - x_1^{(3)} + 2x_3^{(2)}]$$

$$= \frac{1}{3}[7 - 3.3333 + 2 \times 0.8333] = 1.7777$$

$$x_3^{(3)} = \frac{1}{3}[10 - x_1^{(3)} - 2x_2^{(3)}]$$

$$= \frac{1}{3}[10 - 3.3333 - 2 \times 1.7777] = 1.0371$$

$\therefore$ $x_1^{(3)} = 3.3333$ , $x_2^{(3)} = 1.7777$ , $x_3^{(3)} = 1.0371$

$$x_1^{(4)} = \frac{1}{2}[5 + x_2^{(3)} - x_3^{(3)}]$$

$$= \frac{1}{2}[5 + 1.7777 - 1.0371] = 2.8703$$

$x_2^{(4)} = 2.0679$

$x_3^{(4)} = 0.9980$

$\therefore$ $x_1^{(4)} = 2.8703$ , $x_2^{(4)} = 2.0679$ , $x_3^{(4)} = 0.9980$

Now $x_1^{(5)} = 3.035$

$x_2^{(5)} = 1.9870$

$x_3^{(5)} = 0.9970$

$x_1^{(6)} = 2.9950$

$x_2^{(6)} = 1.9997$

$x_3^{(6)} = 1.0019$

$x_1^{(7)} = 2.9989$

$x_2^{(7)} = 2.0016$

$x_3^{(7)} = 0.9993$

$x_1^{(8)} = 3.0011$

$x_2^{(8)} = 1.9991$

$x_3^{(8)} = 1.0002$

$x_1^{(9)} = 2.9994$

$x_2^{(9)} = 2.0003$

$x_3^{(9)} = 1$

$x_1^{(10)} = 3.0001$

$x_2^{(10)} = 1.9999$

$x_3^{(10)} = 1$

$x_1^{(11)} = 2.9999$

$x_2^{(11)} = 2$

$x_3^{(11)} = 1$

$x_1^{(12)} = 3$

$x_2^{(12)} = 2$

$x_3^{(12)} = 1$

$x_1^{(13)} = 3$

$x_2^{(13)} = 2$

$x_3^{(13)} = 1$

Hence the solution of the given system of linear equation is :

$x_1 = 3$ , $x_2 = 2$ , $x_3 = 1$

□ □ □

## Chapter-13

# Forward Difference

**Q.1. Construct a forward difference table for the following given data.**

| | | | | |
|---|---|---|---|---|
| **x** | **3.60** | **3.65** | **3.70** | **3.75** |
| **y** | **36.598** | **38.475** | **40.447** | **42.521** |

**Ans.:**

| x | y | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| 3.60 | 36.598 | 1.877 | 0.095 | 0.007 |
| 3.65 | 38.475 | 1.972 | 0.102 | |
| 3.70 | 40.447 | 2.074 | | |
| 3.75 | 42.521 | | | |

□ □ □

# Chapter-14

# Backward Difference

**Q.1. Construct a backward difference table form the following data :**

**sin 30° = 0.5000, sin 35° = 0.5736, sin 40° = 0.6428, sin 45° = 0.7071**

**Assuming third difference to be constant find the value of sin 25°.**

**Ans.:**

| x | y | $\nabla y$ | $\nabla^2 y$ | $\nabla^3 y$ |
|---|---|---|---|---|
| 25 | ? | | | |
| | | $\nabla y_{30}$ =? | | |
| 30 | 0.5000 | | $\nabla^2 y_{35}$ = ? | |
| | | 0.0736 | | $\nabla^3 y_{40}$ = ? |
| 35 | 0.5736 | | -0.0044 | |
| | | 0.0692 | | -0.0005 |
| 40 | 0.6428 | | -0.0049 | |
| | | 0.0643 | | |
| 45 | 0.7071 | | | |

Since we know that $\nabla^3 y$ should be constant so

$$\nabla^3 y_{40} = -0.0005$$

$$\Rightarrow \nabla^2 y_{40} - \nabla^2 y_{35} = -0.0005$$

$$\Rightarrow -0.0044 - \nabla^2 y_{35} = -0.0005$$

$\nabla^2 y_{35} = +0.0005 - 0.0044$

$= -0.0039$

Again $\nabla^2 y_{35} = -0.0039$

$\nabla y_{35} - \nabla y_{30} = -0.0039$

$\Rightarrow$ $0.0736 - \nabla y_{30} = -0.0039$

$\nabla y_{30} = 0.0039 + 0.0736$

$= 0.0775$

Again $\nabla y_{30} = 0.0775$

$y_{30} - y_{25} = 0.0775$

$\Rightarrow$ $0.5000 - y_{25} = 0.0775$

$y_{25} = 0.5000 - 0.0775$

$= 0.4225$

Hence $\sin 25° = 0.4225$

□ □ □

# Chapter-15

# Newton Gregory Formula for Forward Interpolation

**Q.1. Use Newton formula for interpolation to find the net premium at the age 25 from the table given below :**

| **Age** | **20** | **24** | **28** | **32** |
|---|---|---|---|---|
| **Annual net premium** | **0.01427** | **0.01581** | **0.01772** | **0.01996** |

**Ans.:**

| **Age (x)** | **$f(x)$** | **$\Delta f(x)$** | **$\Delta^2 f(x)$** | **$\Delta^3 f(x)$** |
|---|---|---|---|---|
| 20 | 0.01427 | | | |
| | | 0.00154 | | |
| 24 | 0.01581 | | 0.00037 | |
| | | 0.00191 | | -0.00004 |
| 28 | 0.01772 | | 0.00033 | |
| | | 0.00224 | | |
| 32 | 0.01996 | | | |

Here $a = 20$ , $h = 4$ and $x = a + hu$

$\Rightarrow x = a + hu$

$25 = 20 + 4 \times u$

$5 = 4u \Rightarrow u = 1.25$

Using following Newton's Gregory forward interpolation formula :

$$f(a + hu) = f(a) + u^{(1)}\Delta f(a) + \frac{u^{(2)}}{2!}\Delta^2 f(a) + \frac{u^{(3)}}{3!}\Delta^3 f(a) + \_\_\_\_\_$$

$$\Rightarrow f(25) = 0.01427 + 1.25\ (0.00154) + \frac{1.25\ (0.25)}{1 \times 2}\ (0.00037) + \frac{1.25\ (0.25)\ (-0.75)}{1 \times 2 \times 3}$$
(-0.00004)

$$\Rightarrow f(25) = 0.01427 + 0.001925 + 0.0000578 + 0.0000015 = 0.0162543$$

**Q.2. From the following table find the number of students who obtained less than 45 marks :**

| **Marks** | **No. of Students** |
|---|---|
| **30 – 40** | **31** |
| **40 – 50** | **42** |
| **50 – 60** | **51** |
| **60 – 70** | **35** |
| **70 – 80** | **31** |

**Ans.:**

| **Marks (x)** | **No. of Students $f(x)$** | **$\Delta f(x)$** | **$\Delta^2 f(x)$** | **$\Delta^3 f(x)$** | **$\Delta^4 f(x)$** |
|---|---|---|---|---|---|
| Less than 40 | 31 | | | | |
| | | 42 | | | |
| Less than 50 | 73 | | 9 | | |
| | | 51 | | -25 | |
| Less than 60 | 124 | | -16 | | 37 |
| | | 35 | | 12 | |
| Less than 70 | 159 | | -4 | | |
| | | 31 | | | |
| Less than 80 | 190 | | | | |

Here $a = 40$ , $h = 10$ and $a + hu = 45$

$$\Rightarrow 40 + 10 \times u = 45$$

$$10u = 5$$

$$u = \frac{1}{2}$$

using following forward interpolation formula :

$$f(x) = f(a) + u^{(1)}\Delta f(a) + \frac{u^{(2)}}{2!}\Delta^2 f(a) + \frac{u^{(3)}}{3!}\Delta^3 f(a) + \_\_\_\_\_$$

$$\therefore f(45) = f(40) + \frac{1}{2}\Delta f(40) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\Delta^2 f(40) + \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\Delta^3 f(40)$$

$$+ \frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)\left(\frac{1}{2}-3\right)}{4!}\Delta^4 f(40)$$

$$= 31 + \frac{1}{2}(42) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{1\times 2}(9) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{1\times 2\times 3}(-25) + \frac{\frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{1\times 2\times 3\times 4}(37)$$

$$= 31 + 21 - 1.125 - 1.5625 - 1.4453$$

$$= 47.8672 = 48 \text{ (approximately)}$$

Hence the no. of students who obtained less than 45 marks are 48.

**Q.3. Find the cubic polynomial which takes the following values**

| **x** | **0** | **1** | **2** | **3** |
|---|---|---|---|---|
| **$f$(x)** | **1** | **0** | **1** | **10** |

**Find $f(4)$**

**Ans.:** Here we know that a = 0, h = 1 then form Newton's Gregory forward interpolation formula.

$$P_n(x) = f(0) + {}^{x}c_1\Delta f(0) + {}^{x}c_2\Delta^2 f(0) + \_\ \_\_\ {}^{x}c_n\Delta^n f(0) \qquad \_\_\_\_ (1)$$

| (x) | $f$(x) | $\Delta f$(x) | $\Delta^2 f$(x) | $\Delta^3 f$(x) |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | -1 | | |
| 1 | 0 | | 2 | |
| | | 1 | | 6 |
| 2 | 1 | | 8 | |
| | | 9 | | $f(4) - 27 = 6$ |
| 3 | 10 | | $f(4) - 19$ | (it should be constant) |
| | | $f(4) - 10$ | | |
| 4 | $f(4)$ | | | |

Substituting the values in equation (1) from above table :

$$P_3(x) = 1 + x(-1) + \frac{x(x-1)}{1 \times 2}(2) + \frac{x(x-1)(x-2)}{1 \times 2 \times 3}(6)$$

$$P_3(x) = 1 - x + x^2 - x + x^3 - 3x^2 + 2x$$

$$= x^3 - 2x^2 + 1$$

Hence the required polynomial of degree three is

$$x^3 - 2x^2 + 1$$

Again $f(4) - 27 = 6$

$$\Rightarrow f(4) = 33$$

□ □ □

# Chapter-16

# Newton's Formula for Backward Interpolation

**Q.1. The population of a town in decennial census was as given below :**

| **Year** | **1891** | **1901** | **1911** | **1921** | **1931** |
|---|---|---|---|---|---|
| **Population (in thousands)** | **46** | **66** | **81** | **93** | **101** |

**Estimate the population for the year 1925.**

**Ans.:**

| Year (x) | Population (in thousand) $f(x)$ | $\nabla f(x)$ | $\nabla^2 f(x)$ | $\nabla^3 f(x)$ | $\nabla^4 f(x)$ |
|---|---|---|---|---|---|
| 1891 | 46 | | | | |
| | | 20 | | | |
| 1901 | 66 | | -5 | | |
| | | 15 | | 2 | |
| 1911 | 81 | | -3 | | -3 |
| | | 12 | | -1 | |
| 1921 | 93 | | -4 | | |
| | | 8 | | | |
| 1931 | 101 | | | | |

Here $x = 1925$, $h = 10$ , $a = 1891$ and $a + nh = 1931$

$\therefore$ $(a + hn) + uh = 1925$

$1931 + uh = 1925$

$uh = \dfrac{1925-1931}{10} = -0.6$

Now using Newton's Backward interpolation formula :

$f(a + nh + uh) = f(a + nh) + \dfrac{u}{1!}\nabla f(a + nh) + \_\_\_\_ + \dfrac{u(u+1)(u+2)(u+3)}{4!}$
$\nabla^4 f(a + nh)$

$f(1925) = 101 + (-0.6) \times 8 + \dfrac{(-0.6)(0.4)}{2!}(-4) + \dfrac{(-0.6)(0.4)(1.4)}{3!}(-1)$
$+ \dfrac{(-0.6)(0.4)(1.4)(2.4)}{4!} \times (-3)$

$= 101 - 4.8 + 0.48 + 0.056 - 0.1008$

$= 96.6352$ thousand (approximately)

□ □ □

# Chapter-17

# Divided Difference Interpolation

**Q1. Construct a divided difference table from the following data :**

| **x** | **1** | **2** | **4** | **7** | **12** |
|---|---|---|---|---|---|
| ***f*(x)** | **22** | **30** | **82** | **106** | **216** |

**Ans.:**

| x | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 1 | 22 | | | | |
| | | $\frac{30-22}{2-1} = 8$ | | | |
| 2 | 30 | | $\frac{26-8}{4-1} = 6$ | | |
| | | $\frac{82-30}{4-2} = 26$ | | $\frac{(-3.6-6)}{7-1} = -1.6$ | |
| 4 | 82 | | $\frac{8-26}{7-2} = -3.6$ | | $\frac{0.535-(-1.6)}{12-1} = 0.194$ |
| | | $\frac{106-82}{7-4} = 8$ | | $\frac{1.75-(-3.6)}{12-2} = 0.535$ | |

| x | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 7 | 106 | | $\frac{22-8}{12-4} = 1.75$ | | |
| | | $\frac{216-106}{12-7} = 22$ | | | |
| 12 | 216 | | | | |

**Q.2.** **By means of Newton's divided difference formula find the value of $f(2)$, $f(8)$ and $f(15)$ from the following table :**

| **x** | **4** | **5** | **7** | **10** | **11** | **13** |
|---|---|---|---|---|---|---|
| $f$**(x)** | **48** | **100** | **294** | **900** | **1210** | **2028** |

**Ans.:** Newton's divided difference formula for 4, 5, 7, 10, 11, 13 is :

$$f(x) = f(4) + (x-4)\underset{5}{\Delta} f(4) + (x-4)(x-5)\underset{5,7}{\Delta^2} f(4) + (x-4)(x-5)(x-7)\underset{5,7,10}{\Delta^3} f(4)$$

$$+ (x-4)(x-5)(x-7)(x-10)\underset{5,7,10,11}{\Delta^4} f(4) + \_\_\_\_\_\_\_\_ \qquad \_\_\_ (1)$$

So constructing the following divided difference table :

| x | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 4 | 48 | | | | |
| | | $\frac{100-48}{5-4} = 52$ | | | |
| 5 | 100 | | $\frac{97-52}{7-4} = 15$ | | |
| | | $\frac{294-100}{7-4} = 97$ | | $\frac{21-15}{10-4} = 1$ | |

| x | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ | $\Delta^4 f(x)$ |
|---|---|---|---|---|---|
| 7 | 294 | | $\frac{202-97}{10-5} = 21$ | | 0 |
| | | $\frac{900-294}{10-7} = 202$ | | $\frac{27-21}{11-5} = 1$ | |
| 10 | 900 | | $\frac{310-202}{11-7} = 27$ | | 0 |
| | | $\frac{1210-900}{11-10} = 310$ | | $\frac{33-27}{13-7} = 1$ | |
| 11 | 1210 | | $\frac{409-310}{13-10} = 33$ | | |
| | | $\frac{2028-1210}{13-11} = 409$ | | | |
| 13 | 2028 | | | | |

Substituting the values from above table in equation (1)

$$f(x) = 48 + 52\,(x - 4) + 15\,(x - 4)\,(x - 5) + (x - 4)\,(x - 5)\,(x - 7)$$

$$= x^2\,(x - 1) \qquad \text{\_\_\_(2)}$$

Now substituting x = 2, 8 and 15 in equation (2)

$f(2) = 4\,(2 - 1) = 4$

$f(8) = 64\,(8 - 1) = 448$

$f(15) = 225\,(15 - 1) = 3150$

**Q.3. Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, -21 when x has values 3, 2, 1, -1 respectively.**

**Ans.:** Constructing table according to given data

| x | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| -1 | -21 | | | |
| | | 18 | | |
| 1 | 15 | | -7 | |
| | | -3 | | 1 |
| 2 | 12 | | -3 | |
| | | -9 | | |
| 3 | 3 | | | |

Substituting the values in Newton's divided difference formula :

$f(x) = f(x_0) + (x - x_0)\, f(x_0, x_1) + (x - x_0)(x - x_1) \_\ \_\ \_ (x - x_{n-1}) + f(x_0, x_1, x_2 .. x_n)$

$= -21 + \{x - (-1)\}\, 18 + \{x - (-1)\}(x - 1)(-7) + \{x - (-1)\}(x - 1)(x - 2)(1)$

$= x^3 - 9x^2 + 17x + 6$

□ □ □

## Chapter-18

# Lagrange's Interpolation

**Q.1. Given that**

**$f(1) = 2$ , $f(2) = 4$ , $f(3) = 8$ , $f(4) = 16$ , $f(7) = 128$**

**Find the value of $f(5)$ with the help of Lagrange's interpolation formula.**

**Ans.:** According to question

$x_0 = 1$, $x_1 = 2$, $x_2 = 3$, $x_3 = 4$, $x_4 = 7$, and

$f(x_0) = 2$, $f(x_1) = 4$, $f(x_2) = 8$, $f(x_3) = 16$, and $f(x_4) = 128$,

Using Lagrange's formula for x = 5

$$f(5) = \frac{(5-2)(5-3)(5-4)(5-7)}{(1-2)(1-3)(1-4)(1-7)} \times 2 + \frac{(5-1)(5-3)(5-4)(5-7)}{(2-1)(2-3)(2-4)(2-7)} \times 4$$

$$+ \frac{(5-1)(5-2)(5-4)(5-7)}{(3-1)(3-2)(3-4)(3-7)} \times 8 + \frac{(5-1)(5-2)(5-3)(5-7)}{(4-1)(4-2)(4-3)(4-7)} \times 16$$

$$+ \frac{(5-1)(5-2)(5-3)(5-4)}{(7-1)(7-2)(7-3)(7-4)} \times 128$$

$$= \frac{-2}{3} + \frac{32}{5} - 24 + \frac{128}{3} + \frac{128}{15} = \frac{494}{15}$$

$= 32.93333$

Hence $f(5) = 32.9333$

**Q.2. Find the form of function given by the following table :**

| | | | | |
|---|---|---|---|---|
| **x** | **3** | **2** | **1** | **-1** |
| **$f(x)$** | **3** | **12** | **15** | **-21** |

**Ans.:** According to question

$x_0 = 3,\quad x_1 = 2,\quad x_2 = 1\quad$ and $\quad x_3 = -1$

$f(x_0) = 2,\quad f(x_1) = 12,\quad f(x_2) = 15\quad$ and $\quad f(x_3) = -21$

Now substituting above values in Lagrange's formula :

$$f(x) = \frac{(x-2)(x-1)(x+1)}{(3-2)(3-1)(3+1)} \times 3 + \frac{(x-3)(x-1)(x+1)}{(2-3)(2-1)(2+1)} \times 12$$

$$+ \frac{(x-3)(x-2)(x+1)}{(1-3)(1-2)(1+1)} \times 15 + \frac{(x-3)(x-2)(x-1)}{(-1-3)(-1-2)(-1-1)} \times -21$$

$$= \frac{3}{8}(x^3 - 2x^2 - x + 2) - 4(x^3 - 3x^2 - x + 3) + \frac{15}{4}(x^3 - 4x^2 + x + 6) + \frac{7}{8}$$
$$(x^3 - 6x^2 + 11x - 6)$$

$$f(x) = x^3 - 9x^2 + 17x + 6$$

**Q.3. By means of Lagrange's formula prove that :**

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

**Ans.:** Here we are given $y_{-3}$, $y_{-1}$ $y_1$ and $y_3$ and we have to evaluate $y_0$.

Using Lagrange's formula

$$y_0 = \frac{(0+1)(0-1)(0-3)}{(-3+1)(-3-1)(-3-3)}\, y_{-3} + \frac{(0+3)(0-1)(0-3)}{(-1+3)(-1-1)(-1-3)}\, y_{-1}$$

$$+ \frac{(0+3)(0+1)(0-3)}{(1+3)(1+1)(1-3)}\, y_1 + \frac{(0+3)(0+1)(0-3)}{(3+3)(3+1)(3-1)}\, y_3$$

$$= \frac{-1}{16}\, y_{-3} + \frac{9}{16}\, y_{-1} + \frac{9}{16}\, y_1 - \frac{1}{16}\, y_3$$

$$= \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{16}(y_3 - y_1) - (y_{-1} - y_{-3})$$

$$= \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

Hence proved.

□ □ □

# Chapter-19

# Spline Interpolation

**Q.1. Given the set of data points (1, – 8), (2, – 1) and (3, 18) satisfying the function y = *f*(x). find the linear splines satisfying the given data. Determine the approximate values of y(2.5) and $y^1(2.0)$.**

**Ans.:** Let the given points be A (1, – 8), B (2, – 1) and C (3, 18) equation of AB is

$s_1$ (x) = -8 + (x – 1) 7 [$s_i(x) = y_{i-1} + m_i (x - x_{i-1})$]

= -8 + 7x – 7

= 7x -15

And equation of BC is

$s_2$ (x) = -1 + (x – 2) (19)

= -1 + 19x -38

= 19x – 39 – – – (1)

Since x = 2.5 belongs to the interval [2, 3] we have

y (2.5) = $s_2$ (2.5) = -19(2.5) -39 = 8.5

And $y^1(x)$ = +19 [from equation (1)]

Here we note that the splines $s_i$ (x) are continuous in the interval [1, 3] but their slopes are discontinuous.

□ □ □

# Chapter-20

# Quadratic Splines

**Q.1. Given the set of data points (1, – 8), (2, – 1) and (3, 18) satisfying the function y = *f*(x). find the quadratic splines satisfying the given data. Find also the approximate values of y(2.5) and y' (2.0).**

**Ans.:** Since we know that

$$m_{i-1} + m_i = \frac{2}{h_i}(y_i - y_{i-1}) \qquad [i = 1, 2 \ldots\ldots\ldots\ldots\ldots\ldots n]$$

we have h = 1

taking i = 1

$$m_0 + m_1 = 14$$

taking i = 2

$$m_1 + m_2 = 38$$

Since $m_0 = m_1$ we obtain $m_0 = m_1 = 7$ and $m_2 = 31$ using following equation

$$s_i(x) = \frac{1}{h_i}\left[-\frac{(x_i - x)^2}{2}m_{i-1} + \frac{(x - x_{i-1})^2}{2}m_i\right] + y_{i-1} + \frac{h_i}{2}m_{i-1}$$

$$s_2(x) = -\frac{(x_2 - x)^2}{2}m_1 + \frac{(x - x_1)^2}{2}m_2 + y_1 + \frac{1}{2}m_1$$

$$= -\frac{(3-x)^2}{2}\times(7) + \frac{(x-2)^2}{2}(31) - 1 + \frac{7}{2}$$

$$= -\frac{(3-x)^2}{2}(7) + \frac{31}{2}(x-2)^2 + \frac{5}{2}$$

$= 12x^2 - 41x + 33$

Since 2.5 lies in the interval [2, 3]

Hence

$y\,(2.5) = s_2\,(2.5)$

$= 12(2.5)^2 - 41\,(2.5) + 33$

$= 12 \times 6.25 - 41 \times 2.5 + 33$

$= 5.5$

$y'\,(x) = 24x - 41$

$y'\,(2) = 24 \times 2 - 41$

$= 48 - 41$

$= 7.0$

□ □ □

# Chapter-21

# Cubic Splines

**Q.1. Given the set of data points (1, – 8), (2, – 1) and (3, 18) satisfying the function y = $f$(x). find the cubic splines satisfying the given data. Determine the approximate values of y (2.5) and y' (2.0).**

**Ans.:** We have n = 2 and $p_0 = p_2 = 0$ therefore from the following relation :

$$p_{i-1} + 4p_i + p_{i+1} = \frac{6}{h^2}(y_{i+1} - 2y_i + y_{i-2}) \qquad (i = 1, 2\ldots\ldots n-1)$$

gives

$p_1 = 18$

If $s_1$ (x) and $s_2$ (x) are respectively, the cubic splines in the intervals $1 \le x \le 2$ and $2 \le x \le 3$, we obtain

$s_1 (x) = 3 (x - 1)^3 - 8 (2 - x) - 4 (x - 1)$

and $s_2 (x) = 3 (3 - x)^3 + 22x - 48$

We therefore have

$$y(2.5) = s_2 (2.5) = \frac{3}{8} + 7 = 7.375$$

and $y^1 (2.0) = s_2 (2.0) = 13.0$

□ □ □

# Chapter-22

# Numerical Differentiation

**Q.1. From the following table of values of x and y obtain dy/dx and $d^2y/dx^2$ at x = 1.1.**

| | | | | | | |
|---|---|---|---|---|---|---|
| **x** | **1.0** | **1.2** | **1.4** | **1.6** | **1.8** | **2.0** |
| **y** | **0.00** | **0.1280** | **0.5440** | **1.2960** | **2.4320** | **4.00** |

**Ans.:** According to given question,

h = 0.2, a = 1 and x = 1.1

Here 1.1 is close to the initial value so using Newton-Gregory forward difference formula.

| **x** | **y = *f*(x)** | **Δy** | **$\Delta^2$y** | **$\Delta^3$y** | **$\Delta^4$y** |
|---|---|---|---|---|---|
| 1 | 0.00 | | | | |
| | | 0.1280 | | | |
| 1.2 | 0.1280 | | 0.2880 | | |
| | | 0.4160 | | 0.0480 | |
| 1.4 | 0.5440 | | 0.3360 | | 0 |
| | | 0.7520 | | 0.0480 | |
| 1.6 | 1.2960 | | 0.3840 | | 0 |
| | | 1.1360 | | 0.0480 | |
| 1.8 | 2.4320 | | 0.4320 | | |
| | | 1.5680 | | | |
| 2.0 | 4.0000 | | | | |

Newton's Gregory forward formula is :

$$f(a + xh) = f(a) + {}^{x}c_1\, \Delta f(a) + {}^{x}c_2\, \Delta^2 f(a) + {}^{x}c_3\, \Delta^3 f(a) + \ldots\ldots$$

or $$f(a + xh) = f(a) + x\,\Delta f(a) + \frac{x^2 - x}{2}\,\Delta^2 f(a) + \frac{x^3 - 3x^2 + 2x}{6}\,\Delta^3 f(a) + \ldots \quad \text{---} (1)$$

Differentiating both sides of the equation (1) w. r. t. x

$$hf'(a + xh) = \Delta f(a) + \frac{(2x-1)}{2}\,\Delta^2 f(a) + \frac{3x^2 - 6x + 2}{6}\,\Delta^3 f(a) + \ldots \quad \text{---} (2)$$

Again differentiating equation (2) w. r.t. x

$$h^2 f'(a + xh) = \Delta^2 f(a) + (x - 1)\,\Delta^3 f(a) + \ldots \quad \text{---} (3)$$

Here we have to find f' (1.1) and f'' (1.1)

Substituting a = 1, h = 0.2 and $x = \frac{1}{2}$ in equation (2) and (3)

$$0.2f'(1.1) = 0.1280 + 0 + \frac{1}{6}\left(3 \times \frac{1}{4} - 6 \times \frac{1}{2} + 2\right)(0.0480) + 0$$

Hence f' (1.1) = 0.630 $\quad$ --- (4)

And $\quad (0.2)^2 f''(1.1) = 0.2880 + \left(\frac{1}{2} - 1\right)(0.0480) + 0 = 0.264$

Hence f'' (1.1) = 6.60 $\quad$ --- (5)

**Q.2. Using divided difference find the value of f' (8) given that :**

| **x** | **6** | **7** | **9** | **12** |
|---|---|---|---|---|
| ***f*(x)** | **1.556** | **1.690** | **1.908** | **2.158** |

**Ans.:**

| **x** | **y = *f*(x)** | **Δy** | **Δ²y** | **Δ³y** |
|---|---|---|---|---|
| $x_0 = 6$ | 1.556 | 0.134 | | |

| x | y = $f$(x) | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|
| $x_1 = 7$ | 1.690 | | - 0.0083 | |
| | | 0.109 | | 0.00051 |
| $x_2 = 9$ | 1.908 | | -0.0052 | |
| | | 0.083 | | |
| $x_3 = 12$ | 2.158 | | | |

Newton's divided difference formula is

$f(x) = f(x_0) + (x - x_0)\,\Delta f(x_0) + (x - x_0)(x - x_1)\,\Delta^2 f(x_0) + (x - x_0)(x - x_1)(x - x_2)$
$\Delta^3 f(x_0) + (x + x_0)(x - x_1)(x - x_2)(x - x_3)\,\Delta^4 f(x_0) + \ldots\ldots$ _ _ _ (1)

Differentiating both sides of equation (1) w. r.t. x

$f'(x) = \Delta f(x_0) + (2x - x_0 + x_1)\,\Delta^2 f(x_0) +$

$[3x^2 - 2x(x_0 + x_1 + x_2) + x_0x_1 + x_1x_2 + x_0x_2]\,\Delta^3 f(x_0)$ _ _ _ (2)

Now substituting $x = 8$, $x_0 = 6$, $x_1 = 7$, $x_2 = 9$, $x_3 = 12$ in equation (2)

$f^1(8) = 0.134 + [2 \times 8 - 6 - 7]\,(-0.0083) +$

$[3 \times 64 - 2 \times 8\,(6 + 7 + 9) + 6 \times 7 + 7 \times 9 + 6 \times 9](0.00051)$

$= 0.134 - 0.0249 + (192 - 352 + 159)\,(0.00051)$

$= 0.10859$

□ □ □

# Chapter-23

# Numerical Integration

**Q.1. Compute the value of following integral by Trapezoidal rule.**

$$\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$$

**Ans.:** Dividing the range of integration in equal intervals in the interval [0.2, 1.4]

$$\frac{1.4-0.2}{6} = \frac{1.2}{6} = 0.2 = h$$

| **x** | **sin x** | **$\log_e x$** | **$e^x$** | **y = sin x –log +$e^x$** |
|---|---|---|---|---|
| 0.2 | 0.19867 | -1.6095 | 1.2214 | $y_0$ = 3.0296 |
| 0.4 | 0.3894 | -0.9163 | 1.4918 | $y_1$ = 2.7975 |
| 0.6 | 0.5646 | -0.5108 | 1.8221 | $y_2$ = 2.8975 |
| 0.8 | 0.7174 | -0.2232 | 2.2255 | $y_3$ = 3.1661 |
| 1.0 | 0.8415 | 0.0000 | 2.7183 | $y_4$ = 3.5598 |
| 1.2 | 0.9320 | 0.1823 | 3.3201 | $y_5$ = 4.0698 |
| 1.4 | 0.9855 | 0.3365 | 4.0552 | $y_6$ = 4.7042 |

Using following trapezoidal rule

$$I = \int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$$

$= \frac{h}{2}[(y_0 + y_6) + 2(y_1 + y_2 + y_3 + y_4 + y_5)]$

$= \frac{0.2}{2}[7.7338 + 2(16.4907)]$

$= 4.07152$

**Q.2. Calculate the value of the integral $\int_4^{5.2} \log_e x \, dx$ by Simpson's '$\frac{1}{3}$' rule.**

**Ans.:** First of all dividing the interval [4 5.2] in equal parts.

$\frac{5.2-4}{6} = \frac{1.2}{6} = 0.2 = h$

| $x_i$ | $y_i = \log_e x = \log_{10} x \times 2.30258$ |
|---|---|
| 4.0 | $y_0 = 1.3862944$ |
| 4.2 | $y_1 = 1.4350845$ |
| 4.4 | $y_2 = 1.4816045$ |
| 4.6 | $y_3 = 1.5260563$ |
| 4.8 | $y_4 = 1.5686159$ |
| 5.0 | $y_5 = 1.6049379$ |
| 5.2 | $y_6 = 1.6486586$ |

Using following Simpson's '$\frac{1}{3}$' rule :

$I = \frac{h}{3}[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)]$

$= \frac{0.2}{3}[3.034953 + 18.232315 + 6.1004408]$

$= \frac{0.2}{3}[27.417709] = 1.8278472$

**Q.3.** **Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's '$\frac{3}{8}$' rule :**

**Ans.:** Dividing the interval [0, 1] into six equal intervals.

$$h = \frac{1-0}{6} = \frac{1}{6}$$

| **x** | **y =** $h = \frac{1}{(1+\mathbf{x}^2)}$ |
|---|---|
| $x_0 = 0$ | $y_0 = 1.000$ |
| $x_0 + h = 1/6$ | $y_1 = (36/37) = 0.97297$ |
| $x_0 + 2h = 2/6$ | $y_2 = (36/40) = 0.90000$ |
| $x_0 + 3h = 3/6$ | $y_3 = (36/45) = 0.80000$ |
| $x_0 + 4h = 4/6$ | $y_4 = (36/52) = 0.69231$ |
| $x_0 + 5h = 5/6$ | $y_1 = (36/61) = 0.59016$ |
| $x_0 + 6h = 1$ | $y_6 = (1/2) = 0.50000$ |

Using following Simpson's '3/8' rule.

$$\int_{x_0}^{x_0+nh} ydx = \frac{3h}{8}\ (y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + ......) + 2(y_3 + y_6 + .......)$$

$$\int_0^1 ydx = \frac{1}{16}\ (1 + 0.5) + 3\ (0.97297 + 0.9 + 0.69231 + 0.59016) + 2\ (0.8)$$

$$= \frac{1}{16}\ [1.5 + 9.46632 + 1.6] \quad = 0.785395$$

□ □ □

## Chapter-24

# Numerical Solution for Differential Equations [Euler's Method]

**Q.1. Use Euler's Method to determine an approximate value of y at x = 0.2 from initial value problem $\frac{dx}{dy}$ = 1 – x + 4y y(0) = 1 taking the step size h = 0.1.**

**Ans.:** Here h = 0.1, n = 2, $x_0 = 0$, $y_0 = 1$

Given $\frac{dx}{dy} = 1 - x + 4y$

Hence $y_1 = y_0 + hf(x_0, y_0)$

$= 1 + 0.1\,[1 - x_0 + 4y_0]$

$= 1 + 0.1\,[1 - 0 + 4 \times 1]$

$= 1 + 0.1\,[1 + 4]$

$= 1 + 0.5 \times 5$

$= 1.5$

Similarly $y_2 = y_1 + hf(x_0 + h, y_1)$

$= 1.5 + 0.1[1 - 0.1 + 4 \times 1.5]$

$= 2.19$

**Q.2. Using Euler's Method with step-size 0.1 find the value of y(0.5) from the following differential equation $\frac{dx}{dy} = x^2 + y^2$, y(0) = 0**

**Ans.:** Here h = 0.1, n = 5, $x_0 = 0$, $y_0 = 0$ and $f(x, y) = x^2 + y^2$

Hence $y_1 = y_0 + hf(x_0, y_0)$

$= 0 + (0.1)\,[0^2 + 0^2]$

$= 0$

Similarly $y_2 = y_1 + hf(x_0 + h, y_1)$

$= 0 + (0.1)\,[(0.1)^2 + 0^2]$

$= (0.1)^3$

$= 0.001$

$y_3 = y_2 + hf[x_0 + 2h, y_2]$

$= 0.001 + (0.1)\,[(0.2)^2 + (0.001)^2]$

$= 0.001 + 0.1\,[0.04 + 0.00001]$

$= 0.001 + 0.1\,[0.040001]$

$= 0.005$

$y_4 = y_3 + hf[x_0 + 3h, y_3]$

$= 0.005 + (0.1)\,[(0.3)^2 + (0.005)^2]$

$= 0.005 + (0.1)\,[0.09 + 0.000025]$

$= 0.014$

$y_5 = y_4 + hf[x_0 + 4h, y_4]$

$= 0.014 + (0.1)\,[(0.4)^2 + (0.014)^2]$

$= 0.014 + (0.1)\,[0.16 + 0.00196]$

$= 0.031$

Hence the required solution is 0.031

□ □ □

# Chapter-25

# Numerical Solution for Differential Equations [Euler's Modified Method]

**Q.1. Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$ with initial conditions y = 1 at x = 0 for the range $0 \le x \le 0.6$ in the step of 0.2. Correct upto four place of decimals.**

**Ans.:** Here $f(x, y) = x + |\sqrt{y}|$

$x_0 = 0$, $y_0 = 1$, $h = 0.2$ and $x_n = x_0 + nh$

(i) At x = 0.2

First approximate value of $y_1$

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$
$$= 1 + (0.2)[0 + 1]$$
$$= 1.2$$

Second approximate value of $y_1$

$$y_1^{(2)} = y_0 + \frac{h}{2}[f(x_0, y_0) + f(x_1, y_1^{(1)})]$$
$$= 1 + \frac{0.2}{2}[(0 + 1) + \{0.2 + \sqrt{1.2}\}]$$
$$= 1.2295$$

Third approximate value of $y_1$

$$y_1^{(3)} = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_1, y_1^{(2)})\}$$

$$= 1 + \frac{0.2}{2}\left[(0 + 1) + \{0.2 + \sqrt{1.2295}\}\right]$$

$$= 1 + 0.1\,[1 + 1.30882821]$$

$$= 1.2309$$

Fourth approximate value of $y_1$

$$y_1^{(4)} = y_0 + \frac{h}{2}\{f(x_0, y_0) + f(x_1, y_1^{(3)})\}$$

$$= 1 + \frac{0.2}{2}\left[(0 + 1) + (0.2 + \sqrt{1.2309})\right]$$

$$= 1 + 0.1\,[1 + 1.30945]$$

$$= 1.2309$$

Since the value of $y_1^{(3)}$ and $y_1^{(4)}$ is same

Hence at $x_1 = 0.2$, $y_1 = 1.2309$

(ii) At $x = 0.4$

First approximate value of $y_2$

$$y_2^{(1)} = y_1 + hf(x_1, y_1)$$

$$= 1.2309 + (0.2)\{0.2 + \sqrt{1.2309}\}$$

$$= 1.4927$$

Second approximate value of $y_2$

$$y_2^{(2)} = y_1 + \frac{h}{2}\left[f(x_1, y_1) + f(x_2, y_2^{(1)})\right]$$

$$= 1.2309 + \frac{0.2}{2}\left[(0.2 + \sqrt{1.2309}) + (0.4 + \sqrt{1.4927})\right]$$

$$= 1.2309 + 0.1\,[1.309459328 + (1.621761024]$$

$$= 1.5240$$

Third approximate value of $y_2$

$$y_2^{(3)} = y_1 + \frac{h}{2}\,[f(x_1, y_1) + f(x_2, y_2^{(2)})]$$

$$= 1.2309 + \frac{0.2}{2}\,[(1.309459328 + (0.4 + \sqrt{1.5240}\,)]$$

$$= 1.2309 + 0.1\,[1.309459328 + 1.634503949]$$

$$= 1.5253$$

Fourth approximate value of $y_2$

$$y_2^{(4)} = y_1 + \frac{h}{2}\,\{f(x_1, y_1) + f(x_2, y_2^{(3)})\}$$

$$= 1.2309 + \frac{0.2}{2}\,[(0.2 + \sqrt{1.2309}\,) + (0.4 + \sqrt{1.5253}\,)]$$

$$= 1.2309 + 0.1\,\{1.309459328 + 1.635030364]$$

$$= 1.5253$$

Hence at $x = 0.4$, $y_2 = 1.5253$

(ii) At $x = 0.6$

First approximate value of $y_3$

$$y_3^{(1)} = y_2 + hf(x_2, y_2)$$

$$= 1.5253 + 0.2\,[0.4 + \sqrt{1.5253}\,]$$

$$= 1.8523$$

Second approximate value of $y_3$

$$y_3^{(2)} = y_2 + \frac{h}{2}\,\{f(x_2, y_2) + f(x_3, y_3^{(1)})\}$$

$$= 1.5253 + \frac{0.2}{2}\,[(0.4 + \sqrt{1.5253}\,) + (0.6 + \sqrt{1.8523}\,)]$$

$$= 1.8849$$

Third approximate value of $y_3$

$$y_3^{(3)} = y_2 + \frac{h}{2}\{f(x_2, y_2) + f(x_3, y_3^{(2)})\}$$

$$= 1.5253 + \frac{0.2}{2}[(0.4 + \sqrt{1.5253}) + (0.6 + \sqrt{1.8849})]$$

$$= 1.8851$$

Fourth approximate value of $y_3$

$$y_3^{(4)} = y_2 + \frac{h}{2}\{f(x_2, y_2) + f(x_3, y_3^{(3)})\}$$

$$= 1.5253 + \frac{0.2}{2}[(0.4 + \sqrt{1.5253}) + (0.6 + \sqrt{1.8851})]$$

$$= 1.8851$$

Hence at $x = 0.6$, $y_3 = 1.8851$

□ □ □

## Chapter-26

# Numerical Solution for Differential Equations [Runge – Kutta Method]

**Q.1. Using Runge – Kutta method find an approximate value of y for x = 0.2 in step of 0.1 if $\frac{dy}{dx} = x + y^2$ given y = 1 when x = 0**

**Ans.:** Here $f(x, y) = x + y^2$, $x_0 = 0$, $y_0 = 1$ and $h = 0.1$

$$K_1 = hf(x_0, y_0) = 0.1[0 + 1]$$

$$= 0.1 \ldots\ldots\ldots \qquad \_\_\_ (1)$$

$$K_2 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}K_1)$$

$$= 0.1\left[\left(0 + \frac{1}{2}(0.1)\right) + \left(1 + \frac{1}{2} \times 0.1152\right)^2\right]$$

$$= 0.1152 \qquad \_\_\_ (2)$$

$$K_3 = hf(x_0 + \frac{1}{2}h, y_0 + \frac{1}{2}K_2)$$

$$= 0.1\left[\left(0 + \frac{1}{2}(0.1)\right) + \left\{1 + \left(\frac{1}{2} \times 0.1152\right)\right\}^2\right]$$

$$= 0.1168 \qquad \_\_\_ (3)$$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.1\left[\ 0+0.1\ +\ 1+0.1168\ ^{2}\right]$

$= 0.1347$ _ _ _ (4)

and

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= \frac{1}{6}\ 0.1 + 2(0.1152) + 2(0.1168) + 0.1347$ {using equation (1), (2), (3) and (4)}

$= 0.1165$

Hence $y_1 = y_0 + K = 1 + 0.1165$

$= 1.1165$ _ _ _ (5)

Again $x_1 = x_0 + h = 0.1$, $y_1 = 1.1165$, $h = 0.1$

Now

$K_1 = hf(x_1, y_1)$

$= 0.1\left[0.1 + (1.1165)^2\right]$

$= 0.1347$ _ _ _ (6)

$K_2 = hf\left[x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}K_1\right]$

$= 0.1\left[\left\{0.1 + \frac{1}{2}(0.1)\right\} + \left\{1.1165 + \frac{1}{2}(0.1347)\right\}^2\right]$

$= 0.1551$ _ _ _ (7)

$K_3 = hf\left[x_1 + \frac{1}{2}h, y_1 + \frac{1}{2}K_2\right]$

$= 0.1\left[\left\{0.1 + \frac{1}{2}(0.1)\right\} + \left\{1.1165 + \frac{1}{2}(0.1551)\right\}^2\right]$

$= 0.1576$ _ _ _ (8)

$K_4 = hf\ x_1 + h, y_1 + K_3$

$= (0.1)\left[\ 0.1 + 0.1\ +\ 1.1165 + 0.1576\ ^2\right]$

$= 0.1823$ ——— (9)

and

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= \frac{1}{6}\ 0.1347 + 2(0.1551) + 2(0.1576) + 0.1823$ {using equation (6), (7), (8) and (9)}

$= 0.1570$

Hence

$y(0.2) = y_2 = y_1 + K$

$= 1.1165 + 0.1570$

$= 1.2735$

which is required solution.

**Q.2. Use Runge-Kutta method to solve y' = x y for x = 1.4. Initially x = 1, y = 2 (tale h = 0.2).**

**[BCA Part II, 2007]**

**Ans.:** (i) Here $f(x, y) = xy$, $x_0 = 1$, $y_0 = 2$, $h = 0.2$

$K_1 = hf(x_0, y_0)$

$= 0.2[\ 1 \times 2]$

$= 0.4$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$

$= 0.2\left[\left(1 + \frac{0.2}{2}\right) \times \left(2 + \frac{0.4}{2}\right)\right]$

$= 0.2\left[\ 1+0.1\ \times\ 2+0.2\ \right]$

$= 0.2\left[\ 1.1\quad 2.2\ \right]$

$= 0.484$

$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$

$= 0.2\left[\left(1+\frac{0.2}{2}\right)x\left(2+\frac{0.484}{2}\right)\right]$

$= 0.49324$

$K_4 = hf(x_0 + h, y_0 + K_3)$

$= 0.2\left[\ 1+0.2\ \times\ 2+0.49324\ \right]$

$= 0.5983776$

$K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$= \frac{1}{6}\ 0.4 + 2(0.484) + 2(0.49324) + 0.5983776$

$= 0.4921429$

$y_1 = y_0 + K$

$= 2 + 0.4921429$

$= 2.4921429$

(ii) $x_1 = x_0 + h = 1 + 0.2 = 1.2$, $y_1 = 2.4921429$ and $h = 0.2$

$K_1 = hf(x_1, y_1)$

$= 0.2[(1.2)\ (2.4921429)]$

$= 0.5981143$

$K_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2})$

$$= 0.2\left[\left(1.2+\frac{0.2}{2}\right)\times\left(2.4921+\frac{0.5981143}{2}\right)\right]$$

$= 0.81824$

$$K_3 = hf(x_1+\frac{h}{2}, y_1+\frac{K_2}{2})$$

$$= 0.2\left[\left(1.2+\frac{0.2}{2}\right)\times\left(2.4921+\frac{0.81824}{2}\right)\right]$$

$= 0.7543283$

$K_4 = hf(x_0+h, y_0+K_3)$

$$= 0.2\left[\ 1.2+0.2\ \times\ 2.4921+0.7543\ \right]$$

$= 0.9090119$

$$K = \frac{1}{6}(K_1+2K_2+2K_3+K_4)$$

$= 0.7753$

$y_2 = y_1 + K$

$= 2.4921 + 0.7753$

$= 3.26752$

$\therefore\ y(1.4) = 3.26752$

□ □ □

# Chapter-27

# Boundary Valve Problem - Shooting Method

**Q.1 Solve the Boundary Value Problem y'' (x) = y (x) ; y(0) = 0 ; y(1) = 1.1752 by the shooting method taking $m_0$ = 0.7 and $m_1$ = 0.8**

**Ans.:** By Taylor's Series

$$y(x) = y(0) + xy'(0) + \frac{x^2}{2}y''(0) + \frac{x^3}{6}y'''(0) + \frac{x^4}{24}y^{IV}(0) + \frac{x^5}{120}y^{V}(0) + \frac{x^6}{720}y^{VI}(0) + \ldots \quad \_\_\_ (1)$$

Since $y''(x) = y(x)$ we have

$y'''(x) = y'(x)$

and $y^{IV}(x) = y''(x) = y(x)$

$y^{V}(x) = y'(x)$

$y^{VI}(x) = y''(x) = y(x)$ …………

Putting x = 0 in above we get

$y''(0) = y(0) = 0$ , $y'''(0) = y'(0)$

$y^{IV}(0) = 0$ , $y''(0) = y'(0)$ …………

Substituting these values in equation (1)

$$y(x) = y^{I}(0)\left[x + \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^7}{5040} + \frac{x^9}{362800} + \ldots\ldots\right]$$

Since y(0) = 0

Hence $y(1) = y'(0)\left[x + \frac{1}{6} + \frac{1}{120} + \frac{1}{5040} + \ldots\ldots\ldots\right]$

$= y'(0)(1.1752)$

_ _ _ (2)

With $y'(0) = m_0 = 0.7$

So equation (2) gives

$y(1) \approx 0.8226$

Similarly $y'(0) = m_0 = 0.8$ gives

$y(1) \approx 0.9402$

Using linear interpolation, we obtain

$$m_2 = 0.7 + (0.1)\,\frac{1.1752 - 0.8226}{0.9402 - 0.8226}$$

$= 0.9998$

Which is closer to exact value of $y'(0) = 1$ with this value of $m_2$, we solve the initial value problem $y''(x) = y(x)$, $y(0) = 0$, $y'(0) = m_2$

_ _ _ (3)

and continue the process as above until the value of y(1) is obtained to the desired accuracy.

□ □ □

# Multiple Choice Questions

1. The second divided different of the function f(x) = $\frac{1}{x}$ for the arguments a,b,c is given by:
   (a) $\frac{1}{abc}$
   (b) $-\frac{1}{abc}$
   (c) $abc$
   (d) $-abc$

2. Which method is a successive approximation method, which starts from an approximation to the true solution and, if convergent, the cycles of computations being repeated till the required accuracy is obtained:
   (a) Matrix inversion method
   (b) Matrix factorization method
   (c) Gauss -= elimination method
   (d) Guass-Seidel method

3. If x = 0.555 E01, y= 0.4545 E01 and z=0.4535 E01. Then the value of x (y–z) is equal to:
   (a) 0.5000 E – 01
   (b) 0.555 E –01
   (c) 0.5454 E –01
   (d) 0.5555 E01

4. The logical and concise list of procedure for solving a problem is known as:
   (a) Iterative procedure (b) Approximation method
   (c) Series procedure (d) Algorithm ( )

5. If a positive decimal number be represented in a normalized floating point mode, then the true statement is:
   (a) $0 \leq$ mantissa < 1 (b) $0.1 \leq$ mantissa < 1
   (c) $0.1 \leq$ mantissa < 0 (d) 0.< mantissa $\leq$ 0.1

6. Which of the following stands for divided difference?
   (a) $f(x_0, x) = \frac{f(x_0) - f(x0)}{(x_0 - x)2}$
   (b) $f(x_0, x) = \frac{f(x_0) - f(x)}{x_0 - x^2}$

(c) $f(x_0, x) = \frac{f(x_0) - f(x)}{x_0 - x}$

(d) $f(x_0, x) = \frac{f^2(x_0) - f^2(x)}{(x_0^2 - x^2}$

7. The equation $x^3 - 7x^2 + 8x + 2 = 0$ may have at the most:
   (a) three positive roots
   (b) five positive roots
   (c) two positive roots
   (d) four positive roots

8. The approximate value of y of the solution of:
   $\frac{dy}{dx} = x + y$, when y (0) = 1 by Runge-Kutta method when $k_1 = 0.2000$, $k_2 = 0.2400$, $k_3 = 0.2440$, $k_4 = 0.2888$ is :
   (a) 1.2002
   (b) 1.2428
   (c) 0.2482
   (d) 0.2428

9. The root of the equation $x^3 - 9x+1 = 0$ lies between the interval:
   (a) (1,2)
   (b) (2,3)
   (c) (–1,2)
   (d) (3,4)

10. For the solution of differential equation which of the following methods is not used:
   (a) Runge- Kutta Method
   (b) Shooting method
   (c) Cubic spline method
   (d) Euler's method

11. For the solution of differential equation which of the following methods is not used:
   (a) Runge- Kutta Method
   (b) Shooting method
   (c) Cubic spline method
   (d) Euler's method

12. The approximate value of y of the solution of:
   $\frac{dy}{dx} = x + y$, when y (0) = 1 by Runge-Kutta method when $k_1 = 0.2000$,

$k_2 = 0.2400$, $k_3 = 0.2440$, $k_4 = 0.2888$ is :
(a) 1.2002
(b) 1.2428
(c) 0.2482
(d) 0.2428

13. In which of the following method, pivoting is used:
(a) Euler's method
(b) Gauss Seidel Method
(c) Gause elimination method
(d) Gauss - Jordan Method

14. Numerical integration by Simpson's 1/3 rule is:
(a) Approximation by a parabolic curve
(b) Approximation by a straight line
(c) Approximation by a curve of degree three
(d) Approximation by an elliptic curve

15. The secant methods is:
(a) Modified form of Regula-Falsi method
(b) Modified form of Newton-Raphson method
(c) Modified form of bisection methods
(d) Modified form of Euler methods

16. Truncation error occurs when:
(a) Number is rounding-off during the computation
(b) On replacing infinite process by a finite one
(c) Error already present in the statement of the problem before its solution
(d) None of the above

17. Which of the following is not used in the solution of transcendental equation?
(a) Secant methods
(b) Newton Raphson method
(c) Euler method
(d) Bisection method

18. Which method for the numerical solution of differential equation, is a multi step method?
(a) Euler's method
(b) Runge-Kutta Method
(c) Shooting method

(d) Predictor-corrector method

19. If the imposed conditions that are required to solve a differential equation of higher order are given at more than one-point, then the problem is known is:
(a) Higher value problem
(b) Multi-Value problem
(c) Initial value problem
(d) Boundary value problem ( )

20. In the lagrange's interpolation formula, the sum of lagrangian coefficient is always:
(a) Unity (b) Less than unity
(c) Greater than unity (d) Zero

21. Which interpolation formula cannot have any difference operator?
(a) Stirling's interpolation formula
(b) Bessel's interpolation formula
(c) Newton's genral interpolation formula
(d) Lagrange's interpolation formula

22. In gausses elimination methods, before applying the back substitute process, the system of equations reduces to:
(a) Diagonal matrix
(b) Lower triangular matrix
(c) Upper triangular matrix
(d) Null matrix

23. Which methods for determination of a root of a non-linear equation should never be used when the graph of f (x) is nearly horizontal in the neighborhood of the root ?
(a) Bisection methods
(b) Secant methods
(c) Methods of false position
(d) Newton-Raphson method

24. The equation $x = x - \frac{x-1}{x+1}$ is a :
(a) Linear equation
(b) Non-linear equation
(c) Transcendental equation
(d) None of the above

25. Step by step procedure to solve a problem is knows as:
- (a) Iterative procedure
- (b) Formula
- (c) Technical Procedure
- (d) Algorithm

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (a) | 2. (d) | 3. (b) | 4. (a) | 5. (a) | 6. (c) | 7. (c) | 8. (b) | 9. ( c) | 10. (c) |
| 11. (c) | 12. (b) | 13. (c) | 14. (a) | 15. (a) | 16. (b) | 17. (c) | 18. (d) | 19. (d) | 20.(a) |
| 21. (d) | 22. (c) | 23. (d) | 24. (b) | 25. (a) | | | | | |

# BACHELOR OF COMPUTER APPLICATIONS
# (Part II) EXAMINATION
## (Faculty of Science)
(Three – Year Scheme of 10+2+3 Pattern)
## PAPER 213
# Mathematical Methods for Numerical Analysis and Optimization

## Year - 2011

**Time allowed : One Hour** **Maximum Marks : 20**

***The question paper contains 40 multiple choice questions with four choices and students will have to pick the correct one (each carrying ½ mark).***

1. A large class of techniques is used to provide simultaneous data processing tasks for the purpose of increasing the computational speed of a computer system is known as:
   (a) Series Processing
   (b) Parallel Processing
   (c) Multiple Processing
   (d) Super Processing ( )

2. Which number has greatest absolute error, if each number is correct to the given digits?
   (a) 50.97 (b) 509.7
   (c) 5.097 (d) 0.5097 ( )

3. The number of significant digits in the number 0.0000205000 is:
   (a) 7
   (b) 3
   (c) 10
   (d) 6 ( )

4. Addition of floating point number 0.5723 E05 and 0.2738E07 is equal to:

(a) 0.8461E12
(b) 0.5450E05
(c) 0.2795E07
(d) None of the above ( )

5. Successive approximation method is convergent for the equation x = Ø (x) if:
(a) |Ø (x)| <1
(b) Ø' (x) < 1
(c) |Ø'(x) |< 1
(d) Ø'(x) > 1 ( )

6. The Iterative formula for Newton-Raphson method is:

(a) $x_{n+1} = f(x_n)$

(b) $x_{n+1} = \frac{x_{n-1} f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}$

(c) $x_{n+1} = x_n - \frac{f(x_n)}{f''(x_n)}$

(d) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ ( )

7. Which iterative method has maximum rate of convergence?
(a) Secant method
(b) Bisection method
(c) Regula-falsi method
(d) Newton-Raphson method ( )

8. The equation 2x – log 10x = 7 a :
(a) Transcendental equation
(b) Algebraic Equation
(c) Linear equation
(d) Non-linear equation ( )

9. Which iterative method requires single initial guess root ?
(a) Bisection method
(b) Secant method

(c) Method of false position
(d) Newton Raphson Method ( )

10. If initial guess root of the equation $x^3 – 5x + 3 = 0$ is 1, then first approximation for the root by Newton Raphson method is:
(a) 0.5
(b) 1.5
(c) 1.0
(d) None of the above ( )

11. Which method has slow convergence?
(a) Successive approximation
(b) Secant
(c) Newton-Raphson
(d) Bisection ( )

12. One root of the equation $x^3 + 3x^2 - 5x\ 2 = 0$ lies between:
(a) –5 and –4
(b) –4 and –3
(c) 0 and 1
(d) –1 and +1 ( )

13. For the solution of system of linear equations, in which of the following methods the system of equations reduced to a diagonal system?
(a) Gauss-Seidual method
(b) Jacobi method
(c) Gauss elimination method
(d) Gauss – Jordan elimination method ( )

14. Which method gives approximate values of the variables?
(a) Gauss elimination
(b) Gauss-Jordan
(c) Jocobi
(d) None of the above ( )

15. The first approximate solution by Jacobi method for the system of linear equations:

$$3x + 5y + 10z = 39$$
$$15x + 3y + 7z = 45$$
$$5x + 17y + 8z = 40$$

(a) 13.0, 15.0, 5.0

(b) 3.0, 2.3, 3.9
(c) 8.0, 15.0 3.9
(d) None of the above

16. The technique of estimating unknown value at the point within a set of data values is known as:
(a) Extrapolation
(b) Least square method
(c) Interpolation
(d) None of the above ( )

17. $\Delta^2$y0is equal to:
(a) $y_2 + y_2 - y_0$
(b) $y_2 - y_1 + y_0$
(c) $y_2 + 2_{y1} - y_0$
(d) $y_2 - 2y_1 + y_0$ ( )

18. The values of a function f(x) are given as:

| X | f(x) |
|---|---|
| 0 | 5 |
| 1 | 15 |
| 2 | 25 |
| 3 | 35 |
| 4 | 45 |

The value of f(x) at x = 1.6 is:

(a) 20.5
(b) 21.0
(c) 20.0
(d) None of the above ( )

19. Which interpolation formula is the mean of Gauss's forward and backward formulae?
(a) Newton's Interpolation formula
(b) Stirling's Interpolation formula
(c) Lagrange's Interpolation formula
(d) Bessel's Interpolation formula ( )

20. Which polynomial represents the following tabular values?

x f(x)

| | |
|---|---|
| 0 | 1 |
| 1 | 0 |
| 4 | 45 |
| 5 | 96 |

(a) $x^3 + x^2 - x + 1$
(b) $x^3 - x^2 + x - 1$
(c) $x^3 - x^2 - x + 1$
(d) $x^3 - x^2 + x + 1$ ( )

21. The next term of the sequence 1, 0 1, 10, ………..is:
(a) 11
(b) 33
(c) 22
(d) 44 ( )

22. Which interpolation formula cannot have any differences operators?
(a) Gauss's central differences interpolation formula
(b) Newton's interpolation formula
(c) Lagrange's interpolation formula
(d) None of the above ( )

23. The first derivative of f(x) at x = 0 from the following table is:

| x | f(x) |
|---|---|
| 0 | 4 |
| 1 | 8 |
| 2 | 15 |
| 3 | 7 |
| 4 | 6 |
| 5 | 2 |

(a) –26.7
(b) 26.7
(c) –20.7
(d) 20.7 ( )

24. In Simpson's $\frac{3}{8}$ rule, range of integration is divided into n equal parts, the value of n is:

(a) divisible by 1
(b) divisible by 2
(c) divisible by 6
(d) divisible by 3 ( )

25. The integral $\int_0^{10} x^2 dx$ is evaluated by trapezoidal rule by taking h = 1, the error in the value obtained is:
(a) $-\frac{1}{6}$
(b) $\frac{1}{6}$
(c) $\frac{1}{12}$
(d) $-\frac{1}{12}$ ( )

26. By using Simpson $\frac{1}{3}$ rule, the value of $\int_0^1 e^x$ dx is equal to : (given that e = 2.72, $e^2$ = 7.39, $e^3$ = 20.09, $e^4$ = 54.6)
(a) 51.87
(b) 53.87
(c) 54.87
(d) 52.87 ( )

27. In gauss's three point quadrature formula for weights $w_0$, $w_1$, $w_2$ are as:
a) $\frac{5}{9}, \frac{8}{9}, \frac{5}{9}$
b) $\frac{8}{9}, \frac{5}{9}, \frac{8}{9}$
c) $\frac{5}{9}, \frac{5}{9}, \frac{5}{9}$
d) $\frac{8}{9}, \frac{8}{9}, \frac{8}{9}$ ( )

28. Which of the following formula gives more accurate value of the integral?

(a) Simpson's $\frac{1}{3}$ rule

(b) Trapezoidal rule

(c) Simpson's $\frac{3}{8}$ rule

(d) None of the above ( )

29. The Euler's formula to solve differential equation $\frac{dy}{dx}$ = f (x,y) with y ($x_0$) = $y_0$ is:

(a) $y_{n+1} = y_n \frac{h}{2} f(x_n, y_n)$

(b) $y_{n+1} = y_n\, h\, f\, (x_n, y_n)$

(c) $y_{n+1} = \frac{h}{2} f(x_n, y_n)$

(d) $y_{n+1} = y_n - h\, (x_n, y_n)$ ( )

30. Given $\frac{dy}{dx}$ = x + y with y(0) =1, numerical solution by Runge-Kutta method of fourth order at x = 0.1 (taking h = 0.1) is :

(a) 1.110.34

(b) 0.110.34

(c) 1.10340

(d) 1.11340 ( )

31. In Runge-Kutta method the value of k is given by:

(a) $\frac{1}{6}$ $(k_1+2k_2+k_3+2k_4)$

(b) $\frac{1}{6}$ $(k_1+k_2+k_3+k_4)$

(c) $\frac{1}{6}$ $(k_1+2k_2+2k_3+k_4)$

(d) $\frac{1}{6}$ $(k_1+k_2+2k_3+2k_4)$ ( )

32. Which method is known as multi-step method?
    (a) Euler
    (b) Picards
    (c) Milnes
    (d) Runge-Kutta ( )

33. Given

| X | y |
|---|---|
| 0.01 | 0.1023 |
| 0.02 | 0.1047 |
| 0.03 | 0.1071 |
| 0.04 | 0.1096 |
| 0.05 | 0.1122 |
| 0.06 | 0.1148 |

The value of $\frac{dy}{dx}$ at x = 0.04 will be equal to:
(a) 0.1587 (b) 0.2652
(c) 0.2562 (d) 0.1857 ( )

34. Which of the following is correct?
    (a) $E \equiv 1 + \Delta$ (b) $\Delta \equiv 1 + E$
    (c) $\Delta \equiv 1 - E$ (d) $E \equiv 1 - \Delta$ ( )

35. The operator $(1 + \Delta)(1 - \nabla)$ is equivalent to :
    (a) 2 (b) 1
    (c) –1 (d) 0 ( )

36. Given $\Delta^3 f(10)$ is :
    (a) 0 (b) 1
    (c) 2 (d) 14 ( )

37. For the system 10x + y + z = 12, 2x + 10y + z = 13, 2x +2y + 10z = 14, the value of initial approximation for gauss seidel method are:
    (a) x = 0, y =0, z = 0
    (b) y = 0, z = 0
    (c) x = 1.2, y = 1.3, z = 1.4
    (d) x = 1.2, y =0, z = 0 ( )

38. During Gauss elimination method the elements $a_{ij}$ in the coefficient matrix is known as pivot elements, when:

(a) i = j (b) i < j
(c) i > j (d) I # j ( )

39. When we apply Newton Raphson method to find a root of the equation $x^3–2x–5 = 0$, the first approximation taking $x_0 = 2$ is as:

(a) 2.0 (b) 2.5
(c) 2.25 (d) 1.75 ( )

40. The square root of the number N can be obtained by the following formula:

(a) $x_{n+1} = x_n + \frac{N}{x_n}$

(b) $x_{n+1} = \frac{1}{3}\left[x_n + \frac{N}{x_n}\right]$

(c) $x_{n+1} = \frac{1}{2}\left[x_n - \frac{N}{x_n}\right]$

(d) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{N}{x_n}\right]$ ( )

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. ( ) | 2. ( ) | 3. ( ) | 4.( ) | 5. ( ) | 6. ( ) | 7. ( ) | 8. ( ) | 9. ( ) | 10. ( ) |
| 11. ( ) | 12.( ) | 13. ( ) | 14. ( ) | 15. ( ) | 16. ( ) | 17. ( ) | 18. ( ) | 19. ( ) | 20. ( ) |
| 21. ( ) | 22. ( ) | 23. ( ) | 24. ( ) | 25. ( ) | 26. ( ) | 27. ( ) | 28. ( ) | 29. ( ) | 30. ( ) |
| 31. ( ) | 32. ( ) | 33. ( ) | 34. ( ) | 35. ( ) | 36. ( ) | 37. ( ) | 38. ( ) | 39. ( ) | 40. ( ) |

# DESCRIPTIVE PART – II

## Year 2011

**Time allowed : 2 Hours** **Maximum Marks : 30**

***Attempt any four questions out of the six. All questions carry 7½ marks each.***

1. (a) Use method of iteration to find a real root of the equation: 2x – log 10 x = 7
   (b) Using bisection method, find the real root of the equation $x^4+2x^3-x-1 = 0$ lying in the interval [0,1]

2. (a) Use Newton-Raphson method to find a root of the equation $x^3-2x-5 = 0$
   (b) Use Secant method to find he real root of the equation $x^3-2x-5 = 0$

3. (a) Solve the following system of simultaneous linear equations using gauss elimination method:
   (b) Solve the following system of linear equations using gauss-Seidel iterative method :
   $27x + 6y - z = 85$
   $6x + 15y + 2z = 72$
   $X + y + 54z = 110$

4. (a) Solve the following system linear equations using matrix factorization method:
   $X_1+2x_2 + 3x_3 = 14$
   $2x_1 + 5x_2 + 2x_3 = 18$
   $3x_1 + x_2 + 5x_3 = 20$
   (b) Estimate the missing term in the following table:

| X | f(x) |
|---|---|
| 0 | 1 |
| 1 | 3 |
| 2 | 9 |
| 3 | ? |
| 4 | 81 |

5. (a) The population of a town in the decennial census was as given below. Estimate the population for the year 1895:

| Year | Population (in thousands) |
|---|---|
| 1891 | 46 |
| 1901 | 66 |
| 1911 | 81 |
| 1921 | 93 |
| 1931 | 101 |

(b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using:

(i) Simpson's one third rule

(ii) Simpson's three eight rule.

6. (a) Using Euler's method, find the solution of initial value problem $\frac{dy}{dx}$ = xy, y (0) at x = 0.4, when step-size is 0.1

(b) Using Runge-kutta method, find an approximate value of y for x = 0.2 in steps of 0.1 it $\frac{dy}{dx} = x + y^2$ given y = 1 at x = 0

# Mathematical Methods for Numerical Analysis and Optimination

## Year – 2010

**Time allowed : One Hour** **Maximum Marks : 20**

***The question paper contains 40 multiple choice questions with four choices and students will have to pick the correct one (each carrying ½ mark).***

1. An n-digit floating point number in base $\beta$ of a real number x has the form:
   (a) $\pm$ $(.d_1d_2d_3..................d_n)_\beta\ \beta^e$
   (b) $\pm$ $(.d_1d_2d_3..................d_n)_\beta\ e^\beta$
   (c) $\pm$ $(.d_1d_2d_3..................d_n)_e\ e^\beta$
   (d) $\pm$ $(.d_1d_2d_3..................d_n)_e\ \beta^e$ ( )

2. Truncation error occurs when:
   (a) Number is rounding-off during the computation
   (b) On replacing infinite process by a finite one
   (c) Error already present in the statement of the problem before its solution
   (d) None of the above ( )

3. For the equation $f(x) = 2x^7 - x^5 + 4x^3 - 5 = 0$ which of the following statement is true about its roots:
   (a) Two positive and three negative roots
   (b) More than three positive roots
   (c) At least two complex roots
   (d) More than two negative roots ( )

4. The secant methods is:
   (a) Modified form of Regula-Falsi method
   (b) Modified form of Newton-Raphson method
   (c) Modified form of bisection methods
   (d) Modified form of Euler methods ( )

5. Which of the following is not used in the solution of transcendental equation?
   (a) Secant methods
   (b) Newton Raphson method

(c) Euler method
(d) Bisection method ( )

6. The Newton's iteration formula for the evaluation of $\sqrt{28}$ is:

(a) $x_{n+1} = x_0 - \frac{f(x_0)}{(x_n)}$

(b) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{28}{(x_n)}\right]$

(c) $x_{n+1} = x_n - \frac{x_n - (x_{n-1})}{f(x_n) - f(x_{n-1})}28$

(d) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{(28)^2}{x_n}\right]$ ( )

7. For the solution of system of equations, in which of the following methods the system of equations reduced to upper triangular system:
(a) Gauss – Jorden Method
(b) Gauss–Seidel method
(c) Jacobi method
(d) Gauss elimination method ( )

8. Which of the following is an iteration method for the solution of system of equations:
(a) Cramer'sr rule
(b) Gauss–Jorden method
(c) Gauss – Seidel method
(d) Crout's method ( )

9. The Factorial form of the function $f(x) = 2x^3 - 2x^2 + 3x - 10$ is :
(a) $x^{(3)} - 3x^{(2)} + 2x^{(1)} - 10$
(b) $2x^{(3)} + 3x^{(2)} + 2x^{(1)} - 1$
(c) $2x^{(3)} + 3x^{(2)} + 2x^{(1)} - 10$
(d) $2x^{(3)} - 3x^{(2)} + 2x^{(1)} - 10$ ( )

10. Which of the following is correct:"
(a) $E \equiv 1 + \Delta$
(b) $E^{-1} \equiv 1 - \Delta$

(c) $E^{-1} \equiv 1 - \nabla$

(d) $E \equiv \frac{1}{h} \log (1+\nabla)$ ( )

11. Numerical integration by Simpson's 1/3 rule is:
(a) Approximation by a parabolic curve
(b) Approximation by a straight line
(c) Approximation by a curve of degree three
(d) Approximation by an elliptic curve ( )

12. If there are four arguments $x_0$, $x_0 + h$ , $x_0 + 2h$, $x_0 + 3h$ and corresponding functions values of f (x) are : f ($x_0$), f ($x_0$+h), f ($x_0$+2h), f ($x_0$+3h) given then for the integration of f (x) which of the following method applicable:
(a) Trapezoidal rule
(b) simpson's 1/3 rule
(c) Trapezoidal rula and Simpsons's 1/3 rule
(d) Trapezoidal rule, Simpson's 1/3 rule and Simpson's3 rule ( )

13. For the equation : $\frac{dy}{dx} = f(x, y); y(x_0) = y_0$
then its approximate solution:
$y_n \equiv y_{n+1} + hf(x_0 + \overline{n-1}\, h, y_{n-1})$
is obtained by:
(a) Range-Kutta method
(b) Euler's method
(c) Modified Euler's method
(d) Trapezoidal rule ( )

14. Which of the following statement is incorrect:
(a) Runge-Kutta method do not required the calculation of higher order derivatives
(b) Runge- Kutta method required only the function value at selected points
(c) Euler's method is second order Runge Kutta method
(d) Runge-Kutta method of third order agrees with theh Taylor series solution upto $h^3$ ( )

15. In shooting method, which of the following is correct:
(a) It depends only on initial conditions
(b) It depends on the initial choice of two guesses of the slope m
(c) Its convergence is very high
(d) Easy to apply on h igher order boundary value problem ( )

16. In cubic spline of f(x) which of the following is not correct when data points are : $(x_0,y_0)$, $(x_1,y_1)$,....................$(x_n,y_n)$
(a) f(x) is a linear polynomial outside the interval $(x_0, x_n)$
(b) f (x) is a cubic polynomial in each of the subintervals
(c) f (x) is continuous at each point but continuity of f(x) not essential
(d) f (x) is and f(x) are continuous at each point ( )

17. In gauss formula for n = 2 the value weight and abscissa are:
(a) $w_1 = 0, w_2 = 1, x_1 = -1\ \sqrt{3}, x_2 = 1/\sqrt{3}$
(b) $w_1 = 1, w_2 = 1, x_1 = -1\sqrt{3}, x_2 = 1/\sqrt{3}$
(c) $w_1 = 0, w_2 = 1\ x_1 = -1\sqrt{3}, x_2 = 2/\sqrt{3}$
(d) $w1 = 2, w_2 = 1, x_1 = -1/\sqrt{3}\ x_2 = 1/\sqrt{3}$ ( )

18. $\int f(x)\,dx = \frac{h}{3}(y_0 + y_n)\left\{\begin{array}{l} + 4\ (y_1 + y_3 + \ldots\ldots\ldots\ldots y_{n-1}) \\ + 2\ (y_2 + y_4 + \ldots\ldots\ldots\ldots + y_{n-2}) \end{array}\right\}$
(a) Trapezoidal rule
(b) Simpson's 1/3 rule
(c) Simpson's 3/8 rule
(d) Boolean's rule ( )

19. In which of the following method, pivoting is used:
(a) Euler's method
(b) Gauss Seidel Method
(c) Gause elimination method
(d) Gauss - Jordan Method ( )

20. $\left(\frac{\Delta^2}{E}\right) u_x$ is equal is:
(a) $u_{x+h} + 2u_x + u_{x-h}$
(b) $u_{x+h} - 2u_x + u_{x-h}$
(c) $u_{x+h} + 2u_x + u_{x+h}$
(d) $u_{x+h} - 2u_x + u_{x-h}$ ( )

21. The operation $\Delta - \nabla$ is equivalent to:
(a) $\Delta E$ (b) $\Delta\ \nabla$
(c) $\Delta + \nabla$ (d) $E\Delta^{-1}$ ( )

22. If f (x) = 1/x then divided difference f(a,b) is:
(a) $\frac{ab}{b-c}$ (b) $\frac{-1}{ba}$
(c) $\frac{a-b}{ba}$ (d) –ab ( )

23. Which of the following is true for the divided difference of a function:
(a) Does not depend on the order of the argument
(b) Depends on the order of the argument
(c) $f(x_0, x_1, \ldots\ldots\ldots\ldots x_n) = \frac{h^n}{n!} \Delta^n f(x_0)$
(d) $f(x_0, x_1, \ldots\ldots\ldots\ldots x_n) = \frac{n!}{h^n} \Delta^n f(x_0)$ ( )

24. Newton-Rephson method is:
(a) $x_{n+1} = x_0 - \frac{f(x_0)}{f'(x_n)}$
(b) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$
(c) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
(d) $x_{n+1} = \frac{1}{2}\left[x_n + \frac{f(x_n)}{x_n}\right]$ ( )

25. Which of the following formula can be applied to evaluate:
$\int_0^{1.2} \sin x \, dx$ with step size 0.4 :
(a) Trapezoidal rule
(b) Simpson's 1/3 rule
(c) Trapeaoidal rule and Simpson's 3/8 rule
(d) Simpson's 3/8 rule and Simpson's 1/3 rule ( )

26. The binary equivalent of 0.625 is:
(a) $(0.011)_2$
(b) $(0.101)_2$
(c) $(0.100)_2$
(d) $(0.110)_2$ ( )

27. The decimal equivalent of $(11100111.101)_2$ is:

(a) 231.625
(b) 37.375
(c) 634
(d) 14.875 ( )

28. The root of the equation $x^3 - 9x+1 = 0$ lies between the interval:
(a) (1,2)
(b) (2,3)
(c) (–1,)
(d) (3,4) ( )

29. For the solution of differential equation which of the following methods is not used:
(a) Runge- Kutta Method
(b) Shooting method
(c) Cubic spline method
(d) Euler's method ( )

30. If X is the true value of the quantity and $X_1$ is its approximate value, then: $\frac{X-X1}{X}$ define:
(a) Absolute error
(b) Relative error
(c) Percentage error
(d) Rounding error ( )

31. The equation $2x^3+x^2+3x+10=0$ 0 have:
(a) One real root which is negative
(b) Only one complex root
(c) Three complex root
(d) At the most three positive roots ( )

32. The first approximate value of the solution of:
$\frac{dy}{dx} = x + y$, when y (0) = 1 by Euler method when h = 0.1 is:
(a) 1.2
(b) 1.1
(c) 1.02
(d) 0.01 ( )

33. The approximate value of y of the solution of:

$\frac{dy}{dx}$ = x + y, when y (0) = 1 by Runge-Kutta method when $k_1 = 0.2000$, $k_2 = 0.2400$, $k_3 = 0.2440$, $k_4 = 0.2888$ is :
(a) 1.2002
(b) 1.2428
(c) 0.2482
(d) 0.2428 ( )

34. If 2.1234 be correct to four decimal point then, the error is:
(a) $\frac{1}{2}10^{-3}$
(b) $\frac{1}{2}10^{-5}$
(c) $\frac{1}{2}10^{-4}$
(d) $10^{-3}$ ( )

35. Which of the following is true?
(a) In Regula - Falsi method root is approximated by the tangent to the curve
(b) In Newton-Raphso method root is approximated by a chord
(c) Newton-Raphson method always converges
(d) Regula-Faisi method always converges ( )

36. Which of the following is not true for Newton Raphson method?
(a) It is applicable to the solution of both algebraic and transcendental equation
(b) Its convergence is not depend on the initial guess
(c) It converges is not depend on the initial guess
(d) None of the above ( )

37. Which of the following is known as shift operator?
(a) $\Delta$
(b) E
(c) $\nabla$
(d) $\delta$ ( )

38. The value of $\Delta^3 e^x$ is :
(a) $3!e^x$
(b) $(e-1)^2 e^x$
(c) $(e-1)^3 e^x$

(d) 1/3! $(e-1)^3 e^x$ ( )

39. $\int_{x0}^{x0+4h}$ f (x) dx = $\frac{h}{3}$ [f ($x_0$)+ f ($x_4$)+{f ($x_0$+h)+f($x_0$+3h)}+2 f ($x_2$)] is:

(a) Trapezoidal rule
(b) Simpson's 1/3 rule
(c) Simpson's 3/8 rule
(d) Euler formula ( )

40. For the solution of equations
2x+3y+5z = 1, 2x + 3y +z = 5, 3x+4y+z = 7 by factorization method the value of $u_{11}$, $u_{12}$, $u_{13}$ are:

(a) 1,5,7
(b) 2,2,3
(c) 2,3,5
(d) 1,1,1 ( )

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (d) | 2. (b) | 3. (c) | 4. a) | 5. (c) | 6. (b) | 7. (d) | 8. (c) | 9. (c) | 10. (b) |
| 11. (a) | 12. d) | 13. (b) | 14. (c) | 15. (d) | 16. (c) | 17. (d) | 18. (d) | 19. (c) | 20. (d) |
| 21. (b) | 22. (b) | 23. (a) | 24. (c) | 25. (b) | 26. (b) | 27. (a) | 28. (b) | 29. (c) | 30. (b) |
| 31. (a) | 32. (b) | 33. (b) | 34. (c) | 35. (d) | 36. (b) | 37. (b | 38. (c) | 39. (b) | 40. (c) |

# DESCRIPTIVE PART – II

## Year 2010

**Time allowed : 2 Hours** **Maximum Marks : 30**

***Attempt any four questions out of the six. All questions carry 7½ marks each.***

Q.1 (a) Write the algorithm to find the value of sin x correct to four decimal placs for a givne value of x:

(b) Find real root of $x^3 – 4x – 9 = 0$ correct to three decimal place by Regula-falsi method.

Q.2 (a) Using the starting value $x_0 = i\,(i = \sqrt{1})$ fund a zero of

$$x^4 + x^3 + 5x^2 + 4x + 4 = 0$$

(b) Using factorization method solve:

$$2x_1 + 3x_2 + x_3 = 9$$

$$x_1 + 2x_2 + 3x_3 = 6$$

$$3x_1 + x_2 + 2x_3 = 8$$

Q.3 (a) Using Gauss elimination method, solve the following system of equations:

$$x_1 + x_2 – 6x_3 = –12$$

$$3x_1 – x_2 – x_3 = 4$$

$$x_1 + 4x_2 – x_3 = –5$$

Q.4 (a) Using Newton's dividend difference formula defines f(x) as polynomial in x from the following data:

| x | f(x) |
|---|---|
| –1 | 3 |
| 0 | –6 |

| | |
|---|---|
| 3 | 39 |
| 6 | 822 |
| 7 | 1611 |

(b) For the following data, find the natural cubic splines and evaluate Y (1.5):

| x | y |
|---|---|
| 1 | 1 |
| 2 | 2 |
| 3 | 5 |
| 4 | 11 |

Q.5 (a) Using Simpson's 1/3 rule, find

$$\int_0^5 \frac{dX}{4x+5}$$

(b) Evaluate :

$$\int_{-1}^1 \frac{dX}{1+x2}$$

Using Gauss three point formula.

Q.6 (a) Find y (0.2) and y (0.4) using Runge Kutta fourth order method, from:

$$\frac{dy}{dx} = 1\ y^{2,\ y\ (0)} = 0$$

(b) Using Milne's predictor corrector method find y (0,80 from :

$$\frac{dy}{dx} = 1 + y^{2,}\ y\ (0) = 0$$

given that (0.2) = 0.2027, y (0.4)= 0.4228 and y (0.6) = 0.6841

# Mathematical Methods for Numerical Analysis and Optimization

## Year – 2009

**Time allowed : One Hour** **Maximum Marks : 20**

***The question paper contains 40 multiple choice questions with four choices and students will have to pick the correct one (each carrying ½ mark).***

1. The equation $x^2 + 2 \sin x = 0$ is:
   (a) an algebraic (b) a transcendental equation
   (c) an integral equation (d) a linear equation ( )

2. The quation $x^7+4x^3+2x^2 + 10x +5 = 0$ can be solved by:
   (a) Jacobi method (b) Gauss - Seidel method
   (c) Simpson's 1/3 rule (d) Secant method ( )

3. The equation $x^3 - 7x^2 + 8x + 2 = 0$ may have at the most:
   (a) three positive roots
   (b) five positive roots
   (c) two positive roots
   (d) four positive roots ( )

4. The Newton's Raphson method for the equation f(x) = 0 has initial approximate root $x_0$ . The method fails, if:
   (a) $f'(x_0) = 0$
   (b) $f''(x_0) = 0$
   (c) $f'(x_0) \neq 0$
   (d) $|f''(x_0)| < 1$ ( )

5. The newton raphson method in:
   (a) slower than secant method
   (b) slower than secant method
   (c) faster than secant method
   (d) faster than secant method but slower bisection method ( )

6. The equation f (x = 0 can be written as x = Ø (x). If the equation has $x_0$ as an initial approximation to the root, then the scheme x = Ø (x) will converge to the root of:

(a) $|\phi'(x_0)| > 2$ (b) $|\phi'(x_0)| > 1$

(c) $|\phi(x_0)| < 1$ (d) $|\phi'(x_0)| < 1$ ( )

7. An initial value problem $\frac{dy}{dx}$ = x+ y; y (0) = 5 can be solved by:
(a) Newton Raphson method
(b) Gauss-Seidel method
(c) Simpson's 3/8 rule
(d) None of the above ( )

8. The roots of the equation $x^2 + 4x - 8 = 0$ lies in the interval:
(a) [2,3] (b) [3,4]
(c) [4,5] (d) [0,2] ( )

9. The polynomial equation of n degree has:
(a) (n–1) roots
(b) $n^2$ roots
(c) $\frac{n(n+1)}{2}$ roots
(d) n roots ( )

10. The equation $\frac{dy}{dx} = x^2 + \cos y$, y(0) = 5; x ϵ [0,1] can be solved by:
(a) Shooting method
(b) Jacobi method
(c) Runge-Kutta method
(d) Simpson's 1/3 method ( )

11. The Gauss - Seidel method is applicable only, if:
(a) The system of equations is non-linear
(b) The system of linear equation is diagonally dominant
(c) The system of equations contains three variables
(d) None of the above ( )

12. Which one of the following stands for backward difference operator?

(a) E (b) $\Delta$
(c) $\nabla$ (d) $\delta$ ( )

13. $\int_{x0}^{x0+4h}$ $(where\ h\ is\ step\ size)\ can\ be\ computed\ by$:
(a) Trapezoidal rule not by the Simpson's 1/3 rule
(b) Both Simpson 1/3 and 3/8
(c) Trapezoidal rule and Simpson's 1/3
(d) Simpson 1/3 rule only ( )

14. Lagranges's interpolation formula for equidistant points (arguments) reduces into:
(a) Newton's backword interpolation formula
(b) Newton's forward interpolation formula
(c) Central difference interpolation formula
(d) Newton Raphson formula ( )

15. Which of the following stands for divided difference?

(a) $f(x_0, x) = \frac{f(x_0) - f(x0)}{(x_0 - x)2}$

(b) $f(x_0, x) = \frac{f(x_0) - f(x)}{x_0 - x^2}$

(c) $f(x_0, x) = \frac{f(x_0) - f(x)}{x_0 - x}$

(d) $f(x_0, x) = \frac{f^2(x_0) - f^2(x)}{(x_0^2 - x^2}$ ( )

16. To solve $\int_a^b y$ by gauss quadrature formula, the interval [a,b] is converted to:

(a) [–2,2]

(b) [–2,0]

(c) [–2, –1

(d) [–1,1] ( )

17. The formula 4 $x_{x+1} = x_n - \frac{f(x_{n)}}{f' x_{n)}}$ determines the roots of the equation f (x) = 0 the formula is known as:
(a) Secant method (b) Bisection method
(c) Iterative method (d) Newton raphson method ( )

18. $\int_{x0}^{x0+h}$ y dx = $\frac{h}{3}$ [($y_0$+$y_4$)+4 ($y_1$ + $y_3$)+ 2$y_2$] is:

(a) Trapezoidal rule (b) Simpson rule
(c) Gaussian imeration (d) Exact integration ( )

19. Pivoting is used in solution is:

(a) different equation
(b) integrals
(c) transcendental equation
(d) system of linear equations ( )

20. Runge- kutta method are used for:

(a) Boundary value problem
(b) Initial value problems
(c) Analytical solution
(d) Numerical integration ( )

21. A numerical solution may have:

(a) truncation error (b) propagation error
(c) inherent error (d) all of the above errors ( )

22. If the number 24, 7824621 is represented as 24.782 then the error in this representation is:

(a) Propagation error (b) Round off error
(c) Truncation error (d) Unbounded ( )

23. The Euler method for the IVP $\frac{dy}{dx}$ = f (x,y); y ($x_0$) = $y_0$ is :

(a) $y_{j+1} = y_{j-1} + hf(x_j, y_j)$
(b) $y_{j+1} = y_j + h^2 f(x_j, y_j)$
(c) $y_{j+1} = y_j + hf(x_j, y_j)$
(d) $y_{j+1} = y_j + hf(x_j, y_{j+1})$ ( )

24. Runge-kutta method is:

(a) Single step method (b) Multi step method
(c) Analytic method (d) synthetic method ( )

25. The scant method for the equation f(x) = 0 is:

(a) $x_{n+1} = x_n \frac{f(x_n) - f(x_{n-1})}{(x_n - x_{n-1}}$

(b) $x_{n-1} = x_{n+1} \frac{f(x_n) - f(x_{n-1})}{(x_n - x_{n-1}}$

(c) $x_{n+1} = x_n \frac{f(x_n)}{f'(x_n)}$

(d) None of the above ( )

26. $x^3+7x^2 + 3x-9 = 0$ can be solved by:
(a) Bisection method (b) Newton Raphson Method
(c) Secant Method (d) All of the above method ( )

27. The number of significant digits in the number 0.0082408 is:
(a) Seven
(b) Six
(c) Five
(d) Eight ( )

28. Let X = Exact value; $X_c$ = Computed value, then the absolute error is:

(a) $|X - X_c|$

(b) $\frac{(X-X_C)2}{Xc}$

(c) $\frac{Xc}{(X-X_C)}$

(d) $\left|\frac{X^2-X^2C}{Xc}\right|$ ( )

29. If a = 0.5665 E1; b = 0.5556 E–1, then a+b equals :
(a) 0.5720 E1
(b) 0.5072 E1
(c) 0.5722 E1
(d) 1.0022 E1 ( )

30. If a = 0.9432 E –4; b=0.5452 E–3 then (a–b) equals :
(a) 0.4509 E–9 (b) 0.4666 E3
(c) 0.4509 E3 (d) 0.4508 E–3 ( )

31. The equation $\frac{x^2 y}{dx^2}$ = $x^2$ + $y^2$ ; y (0) = 5; y (2) = 7 is :
(a) an initial value problem
(b) a boundary value problem
(c) Numerical Problem
(d) Unsolvable problem ( )

32. Let $x^2 + 7x - 3 = 0$ be written as
$x = \frac{3-x^2}{7} = \emptyset$ (x)
Let $x_0 = 0.5$ be an initial approximation. The next improved value for the root is:
(a) 0.392
(b) 0.35626
(c) 0.65666
(d) 0.75889 ( )

33. Which of the following is true?
(a) Bisection method may not converge
(b) Newton Raphson method has liner convergence
(c) Bisectioin method has quadratic convergence
(d) None of the above ( )

34. Which of the following is true?
(a) $\Delta \equiv E+1$
(b) $\nabla -1 \equiv E + 5$
(c) $1 + \Delta \equiv E$
(d) $1 - \Delta \equiv E$ ( )

35. Rounding off 4.912575 to five decimals yields:
(a) 4.91257
(b) 4.91125
(c) 4.912576
(d) 4.91258 ( )

36. If the number 5,07932 is approximated to four significant figures, then the absolute error is:
(a) 5.079
(b) 0.7932
(c) 0.00032
(d) 0.0063 ( )

37. The equation f (x) = 0 has toot in [a,b] if:
    (a) f (a) ≠ 0, f (b) ≠ 0
    (b) f (a) < 0, f (b) < 0
    (c) f (a) > 0 f(b) > 0
    (d) f (a) > 0 f (b) < 0 ( )

38. If $x_0 = -1$ $x_1 = -2$ approximations to a root of the equation $x^2 + 4 \sin x = 0$ , then the next approximate value of the root by secant methods is:
    (a) – 1.577632
    (b) –1.60846
    (c) –7.76604
    (d) –1.86704 ( )

39. If h stands for interval of spacing, then $\Delta^2 f(x)$ equals:
    (a) f (x+2h) + f(x)
    (b) f (x+ 2h) –2 f (x + h)
    (c) f (x + 2h) +2 (x+h) + f (x)
    (d) f (x+2h) –2 f (x+h) + f (x) ( )

40. Fourth order Runge Kutta methods uses:
    (a) Two Slopes
    (b) Three slopes
    (c) Four slopes
    (d) Five slopes ( )

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (d) | 2. (b) | 3. (c) | 4. (a) | 5. (c) | 6. (b) | 7. (d) | 8. (c) | 9. (c) | 10. (b) |
| 11. (a) | 12. d) | 13. (b) | 14. (c) | 15. (d) | 16. (c) | 17. (d) | 18. (d) | 19. (c) | 20. (d) |
| 21. (b) | 22. (b) | 23. (a) | 24. (c) | 25. (b) | 26. (b) | 27. (a) | 28. (b) | 29. (c) | 30. (b) |
| 31. (a) | 32. (b) | 33. (b) | 34. (c) | 35. (d) | 36. (b) | 37. (b) | 38. (c) | 39. (b) | 40. (c) |

# DESCRIPTIVE PART – II

## Year 2009

**Time allowed : 2 Hours** **Maximum Marks : 30**

***Attempt any four questions out of the six. All questions carry 7½ marks each.***

Q.1 (a) Compute

$$\frac{x^2-y^2}{x+y}$$

with x = 0.4845 and y = 0 4800, using normalized floating point arithmetic. compare with value (x–y). Determine the relative error of the former.

(b) Find the value $(1+x^2)$when

x=0.5999 E –2

Q.2 (a) Find the root of the equation.

$x^3 - 5x^2 - 17x + 20 = 0$

in the interval [0,1] by secant method.

(b) Use newton Raphson method to find root of the equation $1 + x^3 = \sin x$ in the interval [–1, –2]

Q.3 (a) Solve the system of equations using Gauss elimination method

$5x_1 - 2x_2 + x_3 = 4$

$7x_1 + x_2 - 5x_3 = 8$

$3x_1 + 7x_2 + 4x_3 = 10$

(b) Solve the system of equation using Gauss - Seidel method:

$2x_1 - x_2 + x_3 + 5$

$x_1 + 2x_2 + 3x_3 = 10$

$x_1 + 3x_2 - 2x_2 = 7$

Q.4 (a) Compute f (4), using lagrange's interpolation formula:

| x | f(x) |
|---|---|
| 1.5 | –0.25 |
| 3 | 2 |
| 6 | 20 |

(b) Using the following information, find f (x) as a polynomial in powers of (x–6):

| x | f(x) |
|---|---|
| –1 | –11 |
| 0 | 1 |
| 2 | 1 |
| 3 | 1 |
| 7 | 141 |
| 10 | 561 |

Q.5 (a) Compute the following integral by Simpson's rule using 11 ordinates:

$$\int_0^{\pi/2} \sin x \, dx.$$

(b) Use gauss three point quadrature rule to compute the integral.

$$\int_0^1 \frac{dx}{1+x}$$

Q.6 (a) Using Fourth order Runge-Kutta method with one step. compute (0.1) to five places of decimal, if

$\frac{dy}{dt} = 0.3\, t^2 + 0.25\, y + 0.31$

and y = 0.72 when t = 0

(b) Evaluate y (0.4) by Milne's predictor corrector method where y satisfies differential equation.

$\frac{dy}{dx} = 2e^3 - y$

and the corresponding values of t and y are given as:

| t | y |
|---|---|
| 0 | 2 |
| 0.1 | 2.01 |
| 0.2 | 0.204 |
| 0.3 | 2.09 |

# Mathematical Methods for Numerical Analysis and Optimization

## Year – 2008

**Time allowed : One Hour** **Maximum Marks : 20**

***The question paper contains 40 multiple choice questions with four choices and students will have to pick the correct one (each carrying ½ mark).***

1. Let the true value of a quantity be $\frac{1}{3}$. If we approximate it by the decimal fraction 0.33 then the relative error is equal to:
   (a) $10^{-2}$ (b) $10^{-3}$
   (c) $\frac{1}{3} \times 10^{-2}$ (d) $\frac{1}{3} \times 10^{-3}$ ( )

2. The number of significant digits in the number 0.0001043 is:
   (a) Eight
   (b) Seven
   (c) Four
   (d) Three ( )

3. If a positive decimal number be represented in a normalized floating point mode, then the true statement is:
   (a) $0 \leq$ mantissa < 1 (b) $0.1 \leq$ mantissa < 1
   (c) $0.1 \leq$ mantissa < 0 (d) 0.< mantissa $\leq$ 0.1 ( )

4. If we add the floating point number 0.4123 E 02 and 0.1547 E – 01. Using 4 digit word length, then the result is equal to:
   (a) 0.5670 E01 (b) 0.4124 E 02
   (c) 0.4138 E –1 (d) 0.4138 E 01 ( )

5. The logical and concise list of procedure for solving a problem is known as:
   (a) Iterative procedure (b) Approximation method
   (c) Series procedure (d) Algorithm ( )

6. If x = 0.555 E01, y= 0.4545 E01 and z=0.4535 E01. Then the value of x (y–z) is equal to:
   (a) 0.5000 E – 01

(b) 0.555 E –01
(c) 0.5454 E –01
(d) 0.5555 E01 ( )

7. The equation $x = \frac{5}{x-1}$ is a :
(a) Linear equation
(b) Non-Linear equation
(c) Transcendental
(d) System of equation ( )

8. An iterative formula for a bisection methods is:

(a) $x_{n+1} = x_n \frac{\{f(x_n)-f(x_{n-1})\}}{2}$

(b) $x_{n+1} = x_n - \frac{1}{2}\left\{\frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}}\right\}$

(c) $x_{n+1} = x_n + \frac{1}{2}\left\{\frac{(x_n-x_{n-1})-f(x_n)}{f(x_n)-f(f(x_{n-1})}\right\}$

(d) $x_{n+1} = \frac{x_n+x_{n-1}}{2}$ ( )

9. If 2.5 is the initial root of the equation $x^3–x–10 = 0$, then by the method of Newton Raphson, the next approximate root will be equal to:
(a) 2.3089
(b) 2.55395
(c) 2.6760
(d) 2.6657 ( )

10. If $x_0$ =2.0 and $x_1$ = 3.0 are the two initial roots of the equation $x^3–5x–3 = 0$ then by secant method, the next approximate root $x_2$ will be equal to:
(a) 2.2756
(b) 2.3023
(c) 2.3571
(d) 2.4005 ( )

11. An iterative formula for the method of successive approximation for the equation f (x) = 0 is given by:

(a) $x_{n+1} = x_n - \frac{1}{2}\left\{\frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}}\right\}$

(b) $x_{n+1} = x_n + \frac{1}{2}\left\{\frac{f(x_n)-f(x_{n-1})}{x_n-x_{n-1}}\right\}$

(c) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

(d) $x_{n+1} = \emptyset\,(x_n)$ ( )

12. Which method for determine of root of non linear equation always have guaranteed to converge the required root:
(a) Bisection method
(b) Secant method
(c) Newton Raphson method
(d) Method of successive approximations ( )

13. For the solution of the system of equations, in gauss elimination method, before applying the back substitution process, the given system of equation reduces to:
(a) singular matrix
(b) Diagonal matrix
(c) Lower triangular matrix
(d) Upper triangular matrix ( )

14. The solution of the system of equations by Gauses elimination method is:
2x + 4y + 2z = 15
2x+ y + 2z = –5
4x+y–2z = 0

(a) $-3, 6 - \frac{3}{2}$

(b) $-\frac{55}{18}, \frac{20}{3}, \frac{25}{9}$

(c) $-4, 5, \frac{3}{2}$

(d) $-\frac{35}{18}, \frac{25}{3} \quad \frac{5}{18}$ ( )

15. First approximate solutions by Gauss-Seidel iteration method for the system of equation:
6x+3y+z =9;
2x–5y+2z = –5;
3x+2y+8z= –4

with initial approximation x = 1.0, y = 1,0, Z = –1.5 is:

(a) 1.5, 1.0, –0.5 (b) 1.167, 1.0, – 1.125
(c) 1.167, 1.067, – 1.204 (d) 1.067, 1.125, – 1.204 ( )

16. For the system of equations:
$a_{11}x_1+a_{12}x_2 = a_{13}$ and $a_{21}x_1 + a_{22}\, x_2 = a_{23}$
the gauss seidel iteration method converges it:

(a) $\left|\frac{a12.a21}{a11.a22}\right| > 1$

(b) $\left|\frac{a12.a21}{a11.a22}\right| < 1$

(c) $\left|\frac{a11.a12}{a22.a21}\right| > 1$

(d) $\left|\frac{a11.a12}{a22.a21}\right| < 1$ ( )

17. For the solution of system of the equations:
2x+3y+z = 9
x + 2y + 3z = 6
3x+y+2z = 8
by matrix factorization method by factorized the square matrix a into the form LU, where is the unit lower triangular and U is the upper triangular matrices, then the matrix U is equal to:

(a) $\begin{vmatrix} 1 & 3 & 1 \\ 0 & 1 & 5/2 \\ 0 & 0 & 1 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & -7 & 3/2 \\ 0 & 1 & 1/2 \\ 0 & 0 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 2 & 3 & 1 \\ 0 & 1/2 & 5/2 \\ 0 & 0 & 18 \end{vmatrix}$

(d) $\begin{vmatrix} 2 & -7 & 3/2 \\ 0 & 1/2 & 1/2 \\ 0 & 0 & 9 \end{vmatrix}$ ( )

18. Which method is a successive approximation method, which starts from an approximation to the true solution and, if convergent, the cycles of computations being repeated till the required accuracy is obtained:
(a) Matrix inversion method
(b) Matrix factorization method
(c) Gauss -= elimination method
(d) Guass-Seidel method ( )

19. A set of tabulated valued of x and f (x) for the function y = f (x) are given as:

| x | f (x) |
|---|---|
| 30 | 0.5000 |
| 35 | 0.5736 |
| 40 | 0.6428 |
| 45 | 0.7071 |

then the value of $\nabla^3$ f (x) is equal to:
(a) –0.0004
(b) –0.0005
(c) –0.0006
(d) –0.0044 ( )

20. $\nabla^3 y_1$ is equal to:
(a) $y_3 - 3y_2 + 3y_1 - y_0$
(b) $y_4 - 3y_3 + 3y_2 - y1$
(c) $y_3 + 3y_2 - 3y_1 + y_0$
(d) $y_4 + 3y_3 - 3y_2 + y_1$ ( )

21. Newton's forward difference interpolation formula is useful at:
(a) Interpolation near the beginning of a set of tabular values
(b) Interpolation near the end of a set of tabular values

(c) Interpolation near the middle values of a set of tabular values
(d) Interpolation for unequal arguments at any point ( )

22. Which polynomial represents the following set of value (0,0), (1,3), (2,8), (3,15), (4,24), (5,35):
(a) $x^5 + x^4 + x^3 + x^2 - x$
(b) $x^4 + 2x^3 + x^2 - x$
(c) $2x^3 - x^2 + 2x$
(d) $x^2 + 2x$ ( )

23. The second divided different of the function $f(x) = \frac{1}{x}$ for the arguments a,b,c is given by:
(a) $\frac{1}{abc}$
(b) $-\frac{1}{abc}$
(c) $abc$
(d) $-abc$ ( )

24. Given the following table:

| x | f(x) |
|---|---|
| 1.0 | 0 |
| 1.0 | 0.09531 |
| 1.3 | 0.26236 |

The value of f (x) at x = 1.2 will be equal to:
(a) 0.17268 (b) 0.16827
(c) 0.18276 (d) 0.19346 ( )

25. If arguments are not necessarily be equidistant, then which interpolation formula used to interpolate at any intermediate value of x:
(a) Newton's forward interpolation formula
( b) Newton's backward interpolation formula
(c) Newton's general interpolation formula
(d) Sterling's interpolation formula ( )

26. The piecewise polynomial:

$\Bigg\{$

$$f(x) = \begin{cases} x+1, & -1 \le x \le 0 \\ 2x+1, & 0 \le x \le 1 \\ 4-x, & 1 \le x \le 2 \end{cases}$$

(a) Linear spline (b) Second degree spline
(c) Third degree spline (d) Not a spline ( )

27. The value of the first derivate of f (x) at x = 0.4 from the following table:

| x | f (x) |
|---|---|
| 0.1 | 1.10517 |
| 0.2 | 1.22140 |
| 0.3 | 1.34986 |
| 0.4 | 1.49182 |

( )

is given by:
(a) 1.49182
(b) 1.4913
(c) 1.34986
(d) 1.34762 ( )

28. Which interpolation formula does not have any difference operator:
(a) Newton's general interpolation formula
(b) Bessel's interpolation formula
(c) Stirling's interpolation formula
(d) Lagrange's interpolation formula ( )

29. During the gauss elimination procedure the pivot element ($a_{mk}$) should be searched and the equation with the maximum value of ($a_{mk}$ ) for m = k +1, n should be interchanged with the current equation. This procedure is called:
(a) Maxima Pivoting
(b) Partial Pivoting
(c) Total pivoting
(d) None of these ( )

30. Which numerical integration formula is the trapezoidal rule:
(a) $\int_{x_0}^{x_0+nh} y\,dx = h\left\{\left(\frac{y_0+y_n}{2}\right) + 2\,(y_1 + y_2 + y_3 + \dots\dots + y_{n-1})\right\}$
(b) $\int_{x_0}^{x_0+nh} y\,dx = h\left\{\left(\frac{y_0+y_n}{2}\right) + (y_1 + y_2 + y_3 + \dots\dots + y_{n-1})\right\}$

(c) $\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}$ $\frac{y_0+y_n}{2}$ + ($y_1$+ $y_2$ + $y_3$ + ........ + $y_{n-1}$)

(d) $\int_{x_0}^{x_0+nh} y\,dx = \frac{h}{2}$ $\frac{y_0+y_n}{2}$ + 2 ($y_1$ + $y_2$ + $y_3$ + ........ + $y_{n-1}$) } ( )

31. Using Simpson's 1/3 rule for numerical integration, the value of $\int_0^1 \frac{d}{1+x}$ by taking h = 0.25 will be equal to:
(a) 0.6945
(b) 0.6970
(c) 0.6927
(d) 0.6932 ( )

32. The value of $\int_{0.6}^{0.3} f(x)$ dx, where f (x) is given by the following table, using Simpson's $\frac{3}{8}$ rule is:

| x | f (x) |
|---|---|
| –0.6 | 4 |
| –0.5 | 2 |
| –0.4 | 5 |
| –0.3 | 3 |
| –0.2 | –2 |
| –0.1 | 1 |
| 0 | 6 |
| 0.1 | 4 |
| 0.2 | 2 |
| 0.3 | 8 |

(a) 2.475
(b) 2.457
(c) 2.547
(d) 2.745 ( )

33. If we transform the integral $\int_0^1 x\,dx$ into $\int_1^1 f(u)$ du by gauss quadrature formula, then f (u) is given by:
(a) u+1
(b) $\frac{u+1}{2}$
(c) $\frac{u+1}{4}$

(d) $\frac{u-1}{2}$ ( )

34. If all the imposed conditions that are required to solve a differential we equation are prescribed at one point only, then the differential equation together with the conditions is known as:
    (a) Initial value problem
    (b) Boundary value problem
    (c) Multi value problem
    (d) Higher value problem ( )

35. Which formula is knwon as Euler's method for solution of a differential equation of first order:
    (a) $y_{i+1} = y_i + x_i\, f(x_i, y_i)$
    (c) $y_{i+1} = y_i - x_i\, f(x_i, y_i)$
    (a) $y_{i+1} = y_i - h\, f(x_i, y_i)$
    (c) $y_{i+1} = y_i - \frac{h}{2} f(x_i, y_i)$ ( )

36. Given $\frac{dy}{dx} = 1 - y$ with the initial condition x = 0, Y = 0 then by Euler's method (h = 0.1) the value of y approximately for x = 0.1 is:
    (a) 0
    (b) 0.1
    (c) 0.05
    (d) –0.05 ( )

37. If the solution of a differential equation $\frac{dy}{dx} = f(x, y)$ with $y(x_0) = y_0$ is given by Runge- Kutta fourth order method as $y(x_0 + h) = y_0 + \frac{1}{6}(k_1+2k_2+2k_3+k_4)$, where $k_1$, $k_2$, $k_3$, $k_4$ are Runge kutta coefficients, then the value of $k_4$ is given by :
    (a) $k_4 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_3}{2}\right)$
    (b) $k_4 = hf\left(x_0 - \frac{h}{2}, y_0 - \frac{k_3}{2}\right)$
    (c) $k_4 = hf\left(x_0 + h, y_0 + k_3\right)$
    (d) $k_4 = \frac{h}{4} f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_3}{2}\right)$ ( )

38. The numerical solution at x = 0.1 of the differential equation $\frac{dy}{dx}$ = x+ y with initial condition y (0) = 1 by Runge Kutta Method of fourth order (h = 0.1) is :
(a) 1.11.34
(b) 1.058675
(c) 0.12105
(d) 1.12105 ( )

39. Which method for numerical solution of a differential equation is knwon as 'multi-step method:
(a) Euler's method
(b) Predictor-=corrector method
(c) Runge-Kutta method
(d) None of these ( )

40. 'Shooting method' is a method to solve:
(a) An algebraic equation
(b) A transcedental equation
(c) A differential equation
(d) Both algebraic and transcendental equations ( )

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (a) | 2. (c) | 3. (a) | 4. (b) | 5. (a) | 6. (b) | 7. (b) | 8. (d) | 9. (c) | 10. (c) |
| 11. (d) | 12. (d) | 13. (d) | 14. (b) | 15. (c) | 16. (c) | 17. (c) | 18. (d) | 19. (b) | 20. (b) |
| 21. (a) | 22. (d) | 23. (a) | 24. (c) | 25. (d) | 26. (a) | 27. (b) | 28. (d) | 29. (c) | 30. (a) |
| 31. (d) | 32. (a) | 33. (b) | 34. (a) | 35. (c) | 36. (b) | 37. (c) | 38. (b) | 39. (c) | 40. (c) |

# DESCRIPTIVE PART – II

## Year 2008

**Time allowed : 2 Hours** **Maximum Marks : 30**

***Attempt any four questions out of the six. All questions carry 7½ marks each.***

Q.1 Perform the following arithmetic operations by assuming the mantissa is truncated to six digits. Write the answer in normalized floating point form:

(i) Add the number 0.586351 E05 and 0.9664572 Eo2

(ii) Subtract the number 0.725374 E02 from 0.546332 E03

(iii) Multiply the number 0.654321 E06 and 0.225579 E03

(iv) Divide the number 0.876543 E=05 by 0.246875 E-02

(v) Find the relative error if the number 37.46235 is approximated to four significant figures.

Q.2 (a) Find the root of the equation $x^3 – 5x + 1 = 0$ by disection method, correct up to five decimal places between 0.2016 and 0.2017.

(b) Find the real foot of the equation $x^3–3x – 5 = 0$ correct up to four places of decimals by newton Raphson method.

Q.3 (a) Use the secant method to obtain a root of the equation $x^3 - 5x - 3 = 0$ correct to four decimal places.

(b) Find a root near to zero of the equation $x^3 - 4x + 1 = 0$ by the method of successive approximation correct up to three decimal places.

Q.4 (a) Solve the following system of equation using Gauss elimination method:

$x + 3y + z = 3$

$2x + 3y - 4z = 9$

x + 5y - 2z = 8

(b) Solve the following system of equations using Gauss-Seidel method correct up to three decimal places:

10x+y+2z = 44

2x+10y+z=51

x+2y+10z = 61

or

Solve the following system of equation by matrix factorization method;

5x – 2y + z = 4

7x +y– 5z = 8

3x+7y+4y =10

(b) Matrix factorization method

we have equations

5x - 2y + z = 4

7x + y - 5y

3x+7y + 4y =10

Q.5 (a) Give that;

f(1) = 2, f(2) = 4, f(3) = 8, f(4) = 16, f(7)=128

Find the value of f (5) with help of Lagrange's interpolation formula.

(b) Evaluate the following integral by using gauss - three point quadrature rule;

$\int_0^1 \frac{dx}{1+x}$

Q.6 (a) Using Euler's method with step size 0.1, find the value of y (0.5) from the following differential equation;

$\frac{dy}{dx} = x^2 + y^2$, y (0) = 0

(b) Using Range Kutta method of fourth brder, find an approximate value of y for x = 0.2 in steps of 0.1, if;

$\frac{dy}{dx} = x + y^2$

with y (0) = 1

# Mathematical Methods for Numerical Analysis and Optimization

## Year – 2007

**Time allowed : One Hour** **Maximum Marks : 20**

***The question paper contains 40 multiple choice questions with four choices and students will have to pick the correct one (each carrying ½ mark).***

1. The number of significant digits in the number 0.0003090 is :
   (a) Two (b) Three
   (c) Four (d) Seven ( )

2. If the number $382.00597\times 10^6$ be written in normalized floating point form. Then the exponent part will be :
   (a) $10^6$
   (b) $10^9$
   (c) $10^3$
   (d) $10^8$ ( )

3. If X is the true value of a quantity and X' is its approximate value, then relative error is given by:
   (a) $|x\text{-}x|$
   (b) $\left|\frac{x-x'}{x}\right|$
   (c) $\left|\frac{x}{x-x'}\right|$
   (d) $\left|\frac{x}{x'-x}\right|$ ( )

4. If 0.333 is the approximate value of $\frac{1}{3}$ then the percentage error is equal to:
   (a) 0.0%
   (b) 100.0%

(c) 0.011%
(d) 0.099% ( )

5. Addition of the floating point numbers 0.4546 E05 and 0.5434 E07 is equal to:
(a) 0.9980E05
(b) 0.9980 E07
(c) 0.998 E12
(d) 0.5479E07 ( )

6. If x = 0.5665 E01, y= 0.5556 E–01, and z=0 0.5644 E01, then the value of (x+y) – zis equal is:
(a) 0.7600 E–01
(b) 0.7656 E–01
(c) 0.5577 E–01
(d) 0.5577 E01 ( )

7. A large class of techniques that are used to provide simultaneous does processing tasks for the purpose of increasing the computational speed of a computer system is known is:
(a) Super Processing
(b) Series Processing
(c) Multiple Processing
(d) Parallel Processing ( )

8. The equation x $\frac{x^2-5}{x+9}$ is a :
(a) Linear equation (b) Non-Linear Equation
(c) Transcendental (d) Non-algebraic equation ( )

9. If 0.0, 1.0, 2.0, 3.0 are the initial guess roots of the equation $x^3$–x–1=0, then by Bisection method, the next root will be equal to:
(a) 0.5
(b) 1.5
(c) 2.5
(d) 2.75 ( )

10. If x= 2.3 is the initial root of the equation $x^3$–2x + 5 = 0, then by the method of Newton-Raphson, the next approximate root will be equal to:
(a) –2.1149

(b) –2.2118
(c) –2.0957
(d) 2.2118 ( )

11. An iterative formula for a secant method is:

(a) $X_{n+1} = Xn - \frac{f\,(X_n)}{f'\,(X_n)}$

(b) $X_{n+1} = Xn - \frac{\{f(X_n)-f\,(X_{n+1})\}}{X_n - X_{n-1}}$

(c) $X_{n+1} = Xn + \frac{\{f(X_n)-f\,(X_{n-1})\}}{X_n - X_{n-1}}$

(d) $X_{n+1} = Xn - \frac{(X_n - X_{n-1})\,f\,(X_n)}{f\,(X_n)-f\,(X_{n-1})}$ ( )

12. If $X_0$ = 1.0 and $X_1$ = 2.0 are the two initial roots of the equation $x^3 - 5x + 3 = 0$, then by secant method, the next root $x_2$ will be equal to:
(a) 0.5
(b) 1.5
(c) 0.0
(d) 4.0 ( )

13. Which method having slow sure convergence?
(a) Newton-Raphson Method
(b) Secant Method
(c) Bisection Method
(d) Method of successive approximation ( )

14. During Gauss elimination method, the element, $a_{ij}$ in the coefficient matrix is known as pivot element, when:
(a) i> j
(b) i < j
(c) i= j
(d) $i \neq j$ ( )

15. A method for solution of system of linear equations in which first the system of equation reduce to an equivalent upper triangular system and then solved it by back substitution. This method is known as:
    (a) Matrix seidel method (b) Gauss - Jordan method
    (c) Gauss - Seidel method (d) Gauss elimination method ( )

16. First approximate solution by Gauss-Seidal method for the system of equatm 2, 1
    (b) kbps ( )

17. For the solution of the system of equations:
    $3x + 5y + 2z = 8$; $8y + 2z = -7$ ; $6x + 2y + 8z = 26$
    by matrix factorization method factorize the square matrix A into unit lower triangular and upper triangular matrices L and U, then matrix L is equal is:

    (a) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -1 & 1 \end{vmatrix}$

    (b) $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 1 \end{vmatrix}$

    (c) $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 2 & -1 & 1 \end{vmatrix}$

    (d) $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & -1 & 1 \end{vmatrix}$ ( )

18. Given the set of tabular values $(x_0, y_0)$, $(x_1, y_1)$, $(x_2, y_n)$ satisfying the relation y = f (x) where the explicit natur eo f(x) is not known, it is required to find a simpler function Ø (x) agree at the set of tabulated points, such a process is called;
(a) Iteration
(b) Interpolation
(c) Polynomialization
(d) Factorization ( )

19. A set tabulated values of X and f (x) for the function y = f (x) are given as :

| x | 1 | 1.4 | 1.8 | 2.2 |
|---|---|---|---|---|
| f (x) | 3.49 | 4.82 | 5.96 | 6.5 |

then the value of $\Delta^3$ f(x) is equal is:
(a) –0.19 (b) –0.60
(c) –0.41 (d) 1.33 ( )

20. Newton's backward difference interpolation formula is useful at interpolation:
(a) Near the beginning of a set of tabular values
(b) Near the end of a set of tabular values
(c) Near the middle values of a set of tabular values
(d) For unequal arguments at my point ( )

21. The third divided difference with the arguments a,b,c,d of the function f (x) –1/x is :
(a) abcd (b) –abcd
(c) $\frac{1}{abcd}$ (d) $-\frac{1}{abcd}$ ( )

22. When interpolation formula is merely a relation between two variables x and f(x) either of which may be taken as the independent variable:
(a) Newton's forward interpolation formula
(b) Newton dividend interpolation formula
(c) Lagrange's interpolation formula
(d) Stirling's interpolation formula ( )

23. Estimate f (1) from the following set of values of x and f (x) as:

| x | 0 | 2 | 4 | 6 |
|---|---|---|---|---|
| f (x) | 1 | 1 | 33 | 145 |

(a) 0
(b) 1
(c) 3
(d) 11 ( )

24. Which polynomial represent the following tabular values:

| x | 0 | 1 | 2 | 5 |
|---|---|---|---|---|
| f (x) | 2 | 3 | 12 | 147 |

(a) $x^3 - x^2 + x + 2$
(b) $x^3 + x^2 - x + 2$
(c) $-x^3 + x^2 + x + 2$
(d) $x^3 + 2x^2 - 2x + 2$ ( )

25. Which interpolation formula is the mean by Gauss's forward the backward formula?
(a) Newton's general interpolation formula
(b) Stirling's interpolation formula
(c) Lagrange's interpolation formula
(d) Bessel's interpolation formula ( )

26. Given:

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 4 | 8 | 15 | 7 | 6 |

then the value of $\frac{dy}{dx}$ at x = 0 will be equal to:
(a) –13.5
(b) –27.5
(c) 0
(d) 57.6667 ( )

27. The value of $\frac{dy}{dx}$ f (x) = 0.04 from the following table using Bessel's formula is:

| x | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 |
|---|---|---|---|---|---|---|
| f(x) | 0.1023 | 0.1047 | 0.1071 | 0.1096 | 0.1122 | 0.1148 |

(a) 0.2562

(b) 0.2652
(c) 0.15.87
(d) 0.1857 ( )

28. Which numerical integration formula is the Simpson's $\frac{1}{3}$ rule?

(a) $\int_{x_0}^{x_n} y dx = \frac{h}{3}$ {($y_0$+$y_n$) + 2 ($y_1$+$y_2$+$y_3$+............+$y_{n-1}$)}

(b) $\int_{x_0}^{x_n} y dx = \frac{h}{3}$ {($y_0$+$y_n$) + 3 ($y_1$ +$y_3$ +$y_5$ +............+$y_{n-1}$)

+6($y_2$+$y_4$+$y_6$+..........+$y_{n-2}$)}

(c) $\int_{x_0}^{x_n} y dx = \frac{h}{3}$ {($y_0$+$y_n$) + 2 ($y_1$+$y_3$+$y_5$+............+$y_{n-1}$)}

+ 4 ($y_2$+$y_4$+$y_6$+..........+$y_{n-2}$)}

(d) $\int_{x_0}^{x_n} y dx = \frac{h}{3}$ {($y_0$+$y_n$)+ 4 ($y_1$+$y_3$+$y_5$+............+$y_{n-1}$)}

+2($y_2$+$y_4$+$y_6$+..........+$y_{n-2}$)} ( )

29. Using Trapezoidal Rule with five sub-intervals the value of $\int_0^4$ $e^x$ equal to :
(a) 0.33
(b) 0.25
(c) 0.26
(d) 0.24 ( )

30. By using Simpson's $\frac{1}{3}$ rule, the value of $\int_0^4$ $e^x$ dx equal to (given that e=2.72, $e^2$ = 7.39, $e^3$ 20.09 and $e^4$ = 54.6):
(a) 51.87
(b) 52.87
(c) 53.87
(d) 54.87 ( )

31. If the integral $\int_a^b f(x)$ be transform by Gaussian integration as $\int_1^1 f(u)$ du, then the relation between x and u is given by:
    (a) $u = \left(\frac{b-a}{2}\right)x + \frac{1}{2}(a+b)$
    (b) $u = \left(\frac{a-b}{2}\right)x + \frac{1}{2}(a+b)$
    (c) $x = \left(\frac{b-a}{2}\right)u + \frac{1}{2}(a+b)$
    (d) $x = \left(\frac{a-b}{2}\right)u + \frac{1}{2}(a+b)$ ( )

32. Using Gaussian two point quadrature formula, the value of $\int_2^4 (x^2+2x)\,dx$ is :
    (a) 29.6667
    (b) 30.6667
    (c) 32.6667
    (d) 31.7767 ( )

33. Which of the following numerical integration formula is the best formula?
    (a) Trapezoidal rule
    (b) Simpson's one-third rule
    (c) Simpson's three-eight rule
    (d) None of the above ( )

34. Which of the numerical integration formula does not require equally spaced values of X at which given functional values were used?
    (a) Trapezoidal rule
    (b) Simpson's one - third rule
    (c) Simpson's three - rule
    (d) Gauss's quadrature formula ( )

35. Given $\frac{dy}{dx} = x + y$, with y (0), then by Euler's method, the value of y approximately for x =0.4 (h =0.2) is :
    (a) 0.04
    (b) 0.104
    (c) 0.128
    (d) 0.2736 ( )

36. If the solution of a differential equation (initial value) $\frac{dy}{dx}$ =f (x,y) with y ($x_0$) = $y_0$ is given by Runge-Kutta fourth order method as y ($x_0$+h) = $y_0$ + k, where $k_1$, $k_2$, $k_3$, $k_4$ are Runge Kutta coefficients, then the value of k is given by:

(a) $k=\frac{1}{6}(k_1 + k_2 + k_3 + k_4)$

(b) $k=\frac{1}{6}(k_1 + 2k_2 + k_3 + k_4)$

(c) $k=\frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

(d) $k=\frac{1}{6}(k_1 + 2k_2 + 2k_3 + 2k_4)$ ( )

37. Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y (0) = 1, then the numerical solution by Runge- Kutta method of fourth order at x = 0.2 (take h = 0.2) is:

(a) 1.1414
(b) 1.16678
(c) 1.1837
(d) 1.6662 ( )

38. A method for the numerical solution of a differential equation that in each step uses values from more than one of the preceding steps, is known is:

(a) Single-step method
(b) Multi-step method
(c) Boundary-step method
(d) Higher-step method ( )

39. Given that $\frac{dy}{dx} = 1 + xy^2$, y (0) = 1 with

| | | | |
|---|---|---|---|
| **x** | **0.1** | **0.2** | **0.3** |
| **y** | **1.105** | **1.223** | **1.355** |

then the solution of the differential equation at x= 0.4 Milne's predictor corrector method is equal to:

(a) 0.5396
(b) 1.53380
(c) 1.94481

(d) 1.94462 ( )

40. Which method is used for obtaining the numerical soluation of a boundary value problem:
(a) Euler's method
(b) Runge-Kutta method
(c) Mile's method
(d) Shooting method ( )

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (b) | 2. (b) | 3. (b) | 4. (d) | 5. (d) | 6. (b) | 7. (d) | 8. (a) | 9. (b) | 10. (a) |
| 11. (d) | 12. (b) | 13. (b) | 14. (c) | 15. (d) | 16. (d) | 17. (a) | 18. (b) | 19. (c) | 20. (b) |
| 21. (d) | 22. (c) | 23. (a) | 24. (b) | 25. (b) | 26. (a) | 27. (a) | 28. (d) | 29. (a) | 30. (c) |
| 31. (c) | 32. (b) | 33. (c) | 34. (d) | 35. (a) | 36. (c) | 37. (b) | 38. (b) | 39. (b) | 40. (b) |

# DESCRIPTIVE PART – II

## Year 2007

**Time allowed : 2 Hours** **Maximum Marks : 30**

***Attempt any four questions out of the six. All questions carry 7½ marks each.***

Q.1 (a) If the normalization on floating points is carried out at each stage, prove the following:

(i) $a(b-c) \neq ab-ac$

a = 0.5555 E01

b= 0.4545 E01 and

c = 0.4535 Eo1

(ii) $(a+b)-c \neq (a-c)+b$

a = 0.5665 E01

b = 0.5556 E0–1 and

c = 0.5644 E01

(b) Define absolute, relative and percentage, errors in computer arithmetic. Round off then number 75462 to four significant digits, then calculate the absolute error and percentage errors.

Q.2 (a) Find a real root of the equation $x^3-4x-9=0$ using Bisection method correct to three decimal places.

(b) Using Newton Raphson method, find a real of the equation

$x \log_{10} x = 1.2$

Correct to five decimal places.

Q.3 (a) A real root of the equation $x^3-5x+1=0$ lies in the interval (0,1) perform four iterations of the secant method.

(b) Find a real root of the equation $x^3+x^2-1=0$ by using the methods of successive approximation correct to four decimal places.

Q.4 (a) Solve the following system of equation using Gauss - elimination method:

$2x+y+4z = 12$

$8x-3y + 2z = 20$

$4x + 11y - z = 33$

(b) Solve the following system of equation using the Gauss seidel iteration method correct up to two decimal places.

or

Solve the following system of equations by matrix factorization method:

$3x + 5y+2z = 8$

$8y+2z = -7$

$6x+2y+8z = 26$

Q.5 (a) Using Newton's Divided Difference Formula, find f (8) given f(1) = 3, f (3) = 31, f (6)= 2223, f(6)= 223, f (10) = 1011, f (11) = 1343

(b) Find the first and second derivative of f(x) at x = 0 from the following table:

| **x** | **0** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| **f(x)** | **4** | **8** | **15** | **7** | **6** | **2** |

Q.6 (a) Using Runge-Kutta method of fourth-order solve

$$\frac{dy}{dx} = xy$$

for x = 1.4 Initially x= 1, y =2 take h = 0.2

(b) Solve the boundary value problem
y'' (x) = y (x); y (0)= 0, y(1) = 1.1752
by the shooting method, taking $m_0 = 0.7$ and $m_1 = 0.8$

# Mathematical Methods for Numerical Analysis and Optimization

## Year – 2006

**Time allowed : One Hour** **Maximum Marks : 20**

***The question paper contains 40 multiple choice questions with four choices and students will have to pick the correct one (each carrying ½ mark).***

1. The number is significant digit is the number 0.00018045 is:
   (a) Four
   (b) Five
   (c) Six
   (d) Eight ( )

2. If a positive decimal number be represented in a normalized floating point mode, then the true statement is:
   (a) $1 \leq$ mantissa $\leq 0$
   (b) $0 \leq$ mantissa $< 1$
   (c) $0.1 \leq$ mantissa $< 1$
   (d) $0 <$ mantissa $\leq 0.1$ ( )

3. Normalized floating point form of the number $0.0004382 \times 10^2$ is given by:
   (a) 0.4382E – 01
   (b) 0.4382 E –02
   (c) 0.4382E + 02
   (d) 4.3820 E 01 ( )

4. Addition of the floating point number 0.4123 E02 and 0.1547 E - 01, using 4 digit word lengths is:
   (a) 0.5670 E02
   (b) 0.5670 E–01
   (c) 0.4124 E02
   (d) 0.4124 E–01 ( )

5. Subtraction of 0.9432 E-04 from 0.5452 E- 03 is normalized form is:
   (a) 0.4509 E - 03

(b) 0.45509–04
(c) 0.3980 E – 03
(d) –0.3980 E–04 ( )

6. Step by step procedure to solve a problem is knows as:
(a) Iterative procedure
(b) Formula
(c) Technical Procedure
(d) Algorithm ( )

7. The representation of a finite sequence of simple instructions for solving a problem in a programming language is known is:
(a) Flow chart
(b) Program
(c) Algorithm
(d) Iterative language ( )

8. Errors due to finite representation of an inherently infinite process is knows is:
(a) Rounding errors
(b) Truncation errors
(c) Input errors
(d) Relative errors ( )

9. Round due to finite representation of an inherently infinite process is knows is?
(a) 0.0900
(b) 0.09003
(c) 0.09004
(d) 0.1000 ( )

10. An approximate value of $\pi$ is given by 22/7 = 3.1428571 and its true value is 3.1415926, Then the relative error is:
(a) 0.0012645
(b) 0.00012645
(c) 0.00402
(d) 0.1000 ( )

11. The equation x = $x = \frac{x-1}{x+1}$ is a :
(a) Linear equation
(b) Non-linear equation

(c) Transcendental equation

(d) None of the above ( )

12. If (x) is continuous in a closed interval $a \le x \le b$ and f (a) and f (a), f (b) < 0, then it follows that:

(a) At least one real root of f(x) = 0 lies in the interval a < x < b

(b) No any real root of f(x) = 0 lies in the interval a < x< b

(c) At least one real root of f (x) = 0 lies outside at right of the interval $a \le x \le b$

(d) At least one real root of f (x) = 0 lies outside at left of the interval $a \le x \le b$ ( )

13. If 0, 0.5 and 1.0 are the initial guess roots of the equation $x^3 - 5x + 1 = 0$ then by Bisection method, the next improved root will be equal to:

(a) 0.25 (b) 0.75

(c) 1.25 (d) 0.85 ( )

14. If $X_0 = 2.0$ is the initial root of the equation $x^4 - x - 10 = 0$ then by the method of Newton - Raphson the next approximate root will be equal to:

(a) 1.781

(b) 1.978

(c) 1.871

(d) 1.789 ( )

15. The methods for determination of a root of a non-linear equation which uses two initial guess roots does not require that they must bracket the root is:

(a) Bisection methods

(b) Methods of false position

(c) Secant method

(d) Newton-Raphson method ( )

16. If $x_1 = 3.0$ and $x_2 = 0.0$ are the two initial roots of the equation $x^2 + x - 2 = 0$ they by secant methods, the next root $x_3$ will be equal to:

(a) – 1.0

(b) –2.5

(c) –3.5

(d) –1.5 ( )

17. Which iterative methods requires two real roots?

(a) Bisection method

(b) Secant method

(c) Method of false position

(d) Method of successive approximations ( )

18. Which methods for determination of a root of a non-linear equation should never be used when the graph of f (x) is nearly horizontal in the neighborhood of the root ?
(a) Bisection methods
(b) Secant methods
(c) Methods of false position
(d) Newton-Raphson method ( )

19. The solution of a system of linear equations obtained by direct methods such as gauss elimination or matrix factorization methods, it contains:
(a) Reflection errors
(b) Round-off errors
(c) Data entry errors
(d) Conversation errors ( )

20. In gausses elimination methods, before applying the back substitute process, the system of equations reduces to:
(a) Diagonal matrix
(b) Lower triangular matrix
(c) Upper triangular matrix
(d) Null matrix ( )

21. For the solution of the system of equations, for pivotal condensation may be used along with the methods:
(a) Gauss - seidal iteration methods
(b) Gauss - Elimination methods
(c) Matrix factorization methods
(d) Matrix inverse methods ( )

22. First approximate solutions by Gauss - Seidel methods for the system of equations: $2x + y+z = 5; 3x + 5y+ 2z = 15; 2x + y + 4z = 8$ is:
(a) 0,1.5,.5
(b) 1.5, 1.5, 0.4
(c) 2.5, 1.5, 0.4
(d) 5.0, 3.0, 1.5 ( )

23. In the matrix factorization methods, the matrix A, factorized into L and U, then the system of equation AX = B can be expresses as:
(a) LUX = B

(b) (L+U)X = B
(c) LX = UB
(d) (L–U) X =B ( )

24. For the solution of the system of equations:
3x+2y+z = 10; 3x +3y+2z = 14; x+2y+3z = 14
by matrix factorization methods by factorized the square matrix A into lower and upper triangular matrices L and U, then the the matrix L is equal to:

(a) $\begin{vmatrix} 3 & 3 & 3 \\ 2 & 5/3 & 0 \\ 1 & 4/3 & 24/15 \end{vmatrix}$

(b) $\begin{vmatrix} 1 & 0 & 0 \\ 2/3 & 4/5 & 1 \\ 1/3 & 4/5 & 1 \end{vmatrix}$

(c) $\begin{vmatrix} 1 & 1 & 1 \\ 3 & 1 & 0 \\ 4/3 & 5/3 & 1 \end{vmatrix}$

(d) $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3/4 & 3/5 & 1 \end{vmatrix}$ ( )

25. The missing term in the following tabulated values is:

| x | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| y | 1 | 3 | 9 | – | 81 |

(a) 27 (b) 29
(c) 31 (d) 33 ( )

26. A polynomial of degree two or less which satisface the values (0,1), (1,3), (3,55) will be equal to:
(a) $10x^2–8x+1$ (b) $8x^2-6x+1$
(c) $4x^2–2x+1$ (d) $15x^2–13x+1$ ( )

27. Newton's background difference interpolation formula is useful, when it is required to interpolate:
    (a) Near the beginning of tabulated values
    (b) Near the end of the tabulated values
    (c) Near the central part of the tabulated values
    (d) At any intermediate value of the tabulated values ( )

28. The third divided difference with arguments 2,4,9,10 of the function f (x) = $x^3 - 2x$ will be equal to:
    (a) 26 (b) 15
    (c) 1 (d) –23 ( )

29. an interpolation formula, which is obtained by taking the average of gauses forward and gauss backward interpolation formula, is known is:
    (a) Stirling's interpolation formula
    (b) Bessel's interpolation formula
    (c) Newton general interpolation formula
    (d) Lagrange's interpolation formula ( )

30. Which piecewise polynomial is a spline function:
    (a) $f(x) = \begin{cases} x+1 & -1 \le x \le 0 \\ 2x+1 & 0 \le x \le 1 \\ 4-x & 1 \le x \le 2 \end{cases}$

    (b) $f(x) \begin{cases} x^2+1 & 0 \le x \le 1 \\ 2x^2 & 1 \le x \le 2 \\ 5x-2 & 2 \le x \le 3 \end{cases}$

    (c) $f(x) \begin{cases} x^2 & 0 \le x \le 1 \\ x^2-x+1 & 1 \le x \le 2 \\ 3x-3 & 2 \le x \le 3 \end{cases}$

    (d) None of the above ( )

31. Which interpolation formula cannot have any difference operator?
    (a) Stirling's interpolation formula
    (b) Bessel's interpolation formula
    (c) Newton's genral interpolation formula
    (d) Lagrange's interpolation formula ( )

32. From the following table of values of x and y, the value of dy/dx at x = 5 will be:

| | | | | | |
|---|---|---|---|---|---|
| x | 0 | 2 | 3 | 4 | 7 |
| y | 4 | 26 | 58 | 112 | 466 |

(a) 116 (b) 98
(c) 90 (d) 88 ( )

33. In the lagrange's interpolation formula, the sum of lagrangian coefficient is always:

(a) Unity (b) Less than unity
(c) Greater than unity (d) Zero ( )

34. From the following table, the area bounded by the curve and the x- axis from x = 7.47 to x = 7.52 (apply trapezoidal rule) is:

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 7.47 | 7.48 | 7.49 | 7.50 | 7.51 | 7.52 |
| f (x) | 1.93 | 1.95 | 1.98 | 2.01 | 2.03 | 2.06 |

(a) 0.0498 (b) 0.1992
(c) 0.0996 (d) 0.0249 ( )

35. a river is 80 meters wide. The depth d (in meters) for the river at a distance x from one bank is given by the following table:

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| x | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| y | 0 | 4 | 7 | 9 | 12 | 15 | 14 | 8 | 3 |

Then by Simpson's 1/3 rule, the approximate area of cross section of the river will be equal to:
(a) 700 $m^2$ (b) 705 $m^2$
(c) 710 $m^2$ (d) 805 $m^2$ ( )

36. By Gauses three point quadrature formula, the value of the integral $\int_0^1 \frac{dx}{1+x}$ is equal to:
(a) 0.693122 (b) 0.691322
(c) 0.692231 (d) 0.6901322 ( )

37. If the imposed conditions that are required to solve a differential equation of higher order are given at more than one-point, then the problem is known is:
    (a) Higher value problem
    (b) Multi-Value problem
    (c) Initial value problem
    (d) Boundary value problem ( )

38. Given $\frac{dy}{dx} = x + \sqrt{y}$, with y= 1 for x = 0 then by Euler's modified method (h=0.2) the value of y at x = 0.2 correct upto four places of decimal is:
    (a) 1.2295 (b) 1.23309
    (c) 1.5240 (d) 1.5253 ( )

39. Given $\frac{dy}{dx}$ = xy, with x = 1, y= 2, then the numerical solution, by runge-Kutta method of fourth order at x = 12 (take h = 0.2) is:
    (a) 2.4921429 (b) 2.39121034
    (c) 2.59485866 (d) 2.53933177 ( )

40. Which method for the numerical solution of differential equation, is a multi step method?
    (a) Euler's method
    (b) Runge-Kutta Method
    (c) Shooting method
    (d) Predictor-corrector method ( )

**Answer Key**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1. (b) | 2. (c) | 3. (a) | 4. (c) | 5. (a) | 6. (d) | 7. (a) | 8. (b) | 9. ( c) | 10. (d) |
| 11. (b) | 12. (a) | 13. (a) | 14. (c) | 15. (b) | 16. (a) | 17. (a) | 18. (d) | 19. (b) | 20.(c) |
| 21. (b) | 22. (c) | 23. (a) | 24. (b) | 25. (c) | 26. (b) | 27. (b) | 28. (c) | 29. (a) | 30. (d) |
| 31. (d) | 32. (b) | 33. (a) | 34. (c) | 35. (c) | 36. (b) | 37. (d) | 38. (b) | 39. (a) | 40. (d) |

# DESCRIPTIVE PART – II

## Year 2006

**Time allowed : 2 Hours** **Maximum Marks : 30**

***Attempt any four questions out of the six. All questions carry 7½ marks each.***

Q.1 (a) Discuss the errors that my occur during the floating point arithmetic operations.

(b) Find a real root of the equation $x^3 – x – 4 = 0$ correct to three places of decimal using bisection method.

Q.2 (a) By using newton raphson's method, find the root of $x^4 – x – 10 = 0$ which is near to x=2 correct to three places of decimal.

(b) By using the secant method, find the smallest positive root of the following equation.

$$x^3 – 3x^2 + x + 1 = 0$$

Q.3 (a) Find a smallest positive root of the equation:

$x^3 – 9x + 1 = 0$

by using the method of successive approximation, correct to four decimal places.

(b) Solve the following system of equations by Gauss elimination method

$$3x + 6y + z = 16$$

$$2x + 4y + 3z = 13$$

$$x + 3y + 2z = 9$$

Q.4 (a) Solve the following system of equation by Gauss-Seidel method;

$$x + 3y + z = 10$$

$$x + 2y + 5z = 13$$

$4x+2z=16$

Solve the following system of equations by matrix factorization method:

$$2x+3y+z = 9$$

$$x+2y+3z=6$$

$$3x+y+2z = 8$$

(b) Find the number of men getting wages between Rs. 10 and Rs.15 from the following table:

| **Wages (in Rs.)** | **0-10** | **10-20** | **20-30** | **30-40** |
|---|---|---|---|---|
| **No. of Person** | **9** | **30** | **35** | **42** |

Q.5 (a) By means of Newton's divided difference interpolation formula. Find the vales of f (2) and f (8) from the following table:

| **x** | **4** | **5** | **7** | **10** | **11** | **13** |
|---|---|---|---|---|---|---|
| **f (x)** | **48** | **100** | **294** | **900** | **1210** | **2028** |

(b) Evaluate $\int_0^1 \frac{dx}{1+x^2}$

Q.6 (a) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$, *with* $y = 1$ for $x = 10$

Find y approximately for x =0.1 by Euler's method with step size, h = 0.02.

(b) Solve the equation $\frac{dy}{dx} = x^2 + y$

With initial condition y (0)= 1.0 by Runge-Kutta method of fourth order from x=0.1 to x =0.4 with step length h = 0.1.

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# Keywords

1. Interpolation :- Interpolation is a method of constructing new data points within the range of a discrete set of known data points.
2. Iterative Methods :- In the problems of finding the root of an equation (or a solution of a system of equations), an iterative method uses an initial guess to generate successive approximations to a solution. In contrast, direct methods attempt to solve the problem by a finite sequence of operations.
3. Transcendental Equations : Equations which involve sine, cosine, trigonometric, logarithmic and exponential functions.
4. Numerical Differentiation : numerical differentiation describes algorithms for estimating the derivative of a mathematical function or function subroutine using values of the function and perhaps other knowledge about the function.
5. Or
6. Numerical differentiation is the process of finding the numerical value of a derivative of a given function at a given point. In general, numerical differentiation is more difficult than numerical integration. This is because while numerical integration requires only good continuity properties of the function being integrated, numerical differentiation requires more complicated properties

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