Biyani's Think Tank

Concept based notes

Discrete Mathematics

(MCA I SEM)

Alap Mukherjee
Revised By: Kalpana
BSc (Maths), MCA
Lecturer
Deptt. of Information Technology
Biyani Girls College, Jaipur

Biyani's Group of Girls' Colleges

Published by :
Think Tanks
Biyani Group of Colleges

Sector-3, Vidhyadhar Nagar,
Jaipur-302 023 (Rajasthan)
Ph : 0141-2338371, 2338591-95 • Fax : 0141-2338007
E-mail : acad@biyanicolleges.org
Website :www.gurukpo.com; www.biyanicolleges.org

First Edition : 2011

Leaser Type Setted by :
Biyani College Printing Department

Preface

I am glad to present this book, especially designed to serve the needs of the students. The book has been written keeping in mind the general weakness in understanding the fundamental concepts of the topics. The book is self-explanatory and adopts the “Teach Yourself” style. It is based on question-answer pattern. The language of book is quite easy and understandable based on scientific approach.

Any further improvement in the contents of the book by making corrections, omission and inclusion is keen to be achieved based on suggestions from the readers for which the author shall be obliged.

I acknowledge special thanks to Mr. Rajeev Biyani, *Chairman* & Dr. Sanjay Biyani, *Director* (*Acad.*) Biyani Group of Colleges, who are the backbones and main concept provider and also have been constant source of motivation throughout this Endeavour. They played an active role in coordinating the various stages of this Endeavour and spearheaded the publishing work.

I look forward to receiving valuable suggestions from professors of various educational institutions, other faculty members and students for improvement of the quality of the book. The reader may feel free to send in their comments and suggestions to the under mentioned address.

Author

Syllabus

Introduction to Discrete Mathematical Structures, Formal Methods: Introduction and Analogy, Abstraction.

Fundamentals: Sets & Relations- Sets, Types of Sets, Multi Sets, Operations on Sets, Relations and Properties of Relations, Representation of Relations, Equivalence Relation, Closures of Relations, Methods of Proof-Direct Proofs, Indirect Proofs, Mathematical Induction, Method of Contradiction.

Combinatorics: Permutations and Combinations, Pigeon Hole Principle, Principle of Inclusion and Exclusion, Generating Functions.

Mathematical Logic, Posets and Lattices: Partial Order Set, Bounding Elements, Well Ordered Set, Topological Sorting, Lattices, Principle of Duality, Bounded, Distributed, and Complemented Lattices, Proposition and Propositional Calculus.

Graphs and Group Theory: Basic Introduction of Graphs- Types of Graphs, Path and Circuits,
Eulerian Path and Circuits, Hamiltonian Path and Circuits, Shortest Path Algorithms, Group,
Definitions and Properties, Coset & Subgroup, Normal subgroup,
Homomorphism of groups,Cyclic Group, Permutation Group.

Finite State Machines and Languages: Grammar and Languages- Phrase structure Grammar, Types of Grammars and Languages, Finite State Machines and Languages, Minimization of Finite State Machines.

Contents

Chapter-1

Formal Logic and Propositional Calculus

Q1 What is a Proposition? Give an example.

Ans A Proposition or a statement or logical sentence is a declarative sentence which is either true or false.

For example: 10 > 9 is a proposition because it is always true.

We denote simple propositions, which consist of only one statement by English alphabets like p, q, r, s etc. Each of p, q, r, s can have two values i.e. either true (T) or false(F). Hence p, q, r, s are called **propositional variables.**

True and False are called **propositional constants.**

Q2 What is a Predicate? Give appropriate example.

Ans All that is told about the subject in a sentence is called predicate.

For example:- "***Amit is a good boy***".

Here ' ***is a good boy*** ' is the predicate and '***Amit*** ' is subject.

Let ' ***is a good boy*** ' is denoted by ***'B'*** and '***Amit*** 'is denoted by ' ***J*** ' then the above sentence can be written in logical form as ***B(J).***

<u>**Note:-**</u> If Q is predicate then sentence 'P is Q' is denoted by Q(P).

Q3 What is a compound statement? List the logical connectives.

Ans Statements or propositional variables can be combined by means of logical connectives (operators) to form a single statement called compound statements.

The five logical connectives are:

Symbol	Connective	Name
~	Not	Negation
∧	And	Conjunction
∨	Or	Disjunction
→	Implies or if…then	Implication or conditional
↔	If and only if	Equivalence or biconditional

Q4 What is the negation of a statement? Give example.

Ans If p is a statement then negation of p is denoted by ~p and read as 'it is not the case that p'. So, if p is true then ~p is false and vice versa.
Example: if statement p is Paris is in France , then ~p is 'Paris is not in France'.

p	~p
T	F
F	T

Q5 What do you mean by the conjunction of two statements? Give examples.

Ans If p , q are two statements, then "p and q" is a compound statement, denoted by p ∧ q and referred as the conjunction of p and q.
The conjunction of p and q is true only when both p and q are true, otherwise it is false.

p	**q**	**p ∧ q**
T	T	T
T	F	F
F	T	F
F	F	F

Example: if statement p is "6<7" and statement q is "-3>-4" then the conjunction of p and q is true as both p and q are true statements.

Q6 What do you mean by the disjunction of two statements? Give examples.

Ans If p , q are two statements, then "p or q" is a compound statement, denoted by p ∨ q and referred as the disjunction of p and q.

The disjunction of p and q is true whenever at least one of the two statements is true, and it is false only when both p and q are false.

p	**q**	**p ∨ q**
T	T	T
T	F	T
F	T	T
F	F	F

Example:- if p is "4 is a positive integer" and q is "$\sqrt{5}$ is a rational number", then p ∨ q is true as statement p is true, although statement q is false.

Q7 What do you mean by the Conditional Statement (or Implication) of two statements? Give examples.

Ans Let p and q are two statements then "if p then q" is a compound statement, denoted by p → q and referred as a conditional statement, or implication.The implication p → q is false only when p is true and q is false; otherwise it is always true.

In this implication, p is called the hypothesis(or antecedent) and q is called the conclusion(or consequent).

p	Q	p → q
T	T	T
T	F	F
F	T	T
F	F	T

Q8 What are the Converse, Contrapositive and Inverse of a conditional statement? Give the Truth Tables.

Ans If p → q is an implication, then the Converse of p → q is "q → p".

The Contrapositive of p → q is the implication " ~q → ~p ".

The Inverse of p → q is the implication "~p → ~q ".

p	q	$p \to q$	$q \to p$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Table1) Truth Table of $q \to p$(converse)

p	q	~p	~q	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Table2) Truth Table of $\sim q \to \sim p$ (Contrapositive)

p	q	~p	~q	$p \to q$	$\sim p \to \sim q$
T	T	F	F	T	T
T	F	F	T	F	T
F	T	T	F	T	F
F	F	T	T	T	T

Table3) Truth Table of $\sim p \to \sim q$ (Inverse)

Q9 Find the Converse, Contrapositive and Inverse of the following implications:-
(i) "If today is Thursday, then I have a test today"

Ans Let p : "Today is Thursday" and q : "I have a test today"
Then Converse $q \to p$ is "If I have a test today, then today is Thursday"
Contrapositive $\sim q \to \sim p$ is "If I do not have a test today, then today is not Thursday".
Inverse $\sim p \to \sim q$ is "If today is not Thursday, then I do not have a test today".

Q10 What is a Biconditional statement or Equivalence ?

Ans If p and q are two statements then "p if and only if q" is a compound statement, denoted as $p \leftrightarrow q$ and reffered as a biconditional statement or an equivalence.

The equivalence $p \leftrightarrow q$ is true only when both p and q are true or when both p and q are false.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Q11 Is the following equivalence a true statement ?
"3 > 2 if and only if 0 < 3 - 2"

Ans Let p: 3 >2 and q : 0 <3 – 2
Clearly both p and q are true, thus $p \leftrightarrow q$ is also true.

Q12 Explain the Direct Proof approach.

Ans We will explain the direct proof approach using following example:
Let the following statements be true.

1) It is snowing.
2) If it is warm, then it is not snowing.
3) If it is not warm, then I cannot go for swimming.

Show that the statement ' I cannot go for swimming ' is a true statement.
Proof) :- let p, q, r represents the statements
P : It is snowing
Q : It is warm
R : I can go for swimming.
Consider the premises that the statements P, $Q \Rightarrow \sim P$, $\sim Q \Rightarrow \sim R$ are true.
Then we have to prove that:
$\sim R$: 'I cannot go for swimming is true'.
Since, P is true and $Q \Rightarrow \sim P$, so its contrapositive $\sim(\sim P) \Rightarrow \sim Q$ i.e. $P \Rightarrow \sim Q$ is also true.
But $\sim Q \Rightarrow \sim R$ is true
Therefore $P \Rightarrow \sim Q$ and $\sim Q \Rightarrow \sim R$. So by law of syllogism we say
$P \Rightarrow \sim R$
And because P is true, $\sim R$ must also be true.

Q13 Explain the Indirect Proof approach.

Ans We will explain this with the help of an example:

Example: Show that $5 + \sqrt{2}$ is irrational number.

Proof) :- let $5 + \sqrt{2} = r$, is a rational number or $\sqrt{2} = r - 5$

Since r and 5 are rational number so $(r - 5)$ is also rational.

This implies that $\sqrt{2}$ is also rational, which is contradiction because $\sqrt{2}$ is irrational number.

Thus the assumption that $5 + \sqrt{2}$ is rational number is false, hence the given number is irrational.

Chapter-2

Sets

Q1 Define a set giving examples?

Ans A well defined collection of objects or elements is called a set. Each of these elements has to satisfy a property by which it can be decided whether a given object belongs to the set or not.

Examples of sets are:

a)A set of rivers of India.

b)A set of vowels.

We generally denote a set by capital letter A, B, C etc. while the elements of the set by small letter a, b, x, y etc.

If A is a set , and a is one of the element of A, then we denote it as $a \in A$. Here the symbol $\in$ denotes "Element of".

Q2 How are sets represented? Give appropriate examples.

Ans Sets are represented in two forms:-

a) Roster or tabular form: In this from of representation we list all the elements of the set within braces { } and separate them by commas.
Example: if A= set of all odd numbers less then 10 then in roster from it can be written as A={ 1,3,5,7,9}.

b) Set Builder form: In this form of representation we list the properties satisfied by all the elements of the set. We write as { x: x satisfies properties P}. and read as ' the set of all those x such that each x has properties P '.
Example: if B= {2,4,8,16,32}, then the set builder representation will be:
$B=\{ x: x=2^n$, where $n \in N$ and $1 \leq n \geq 5$ }

Q3 What are Finite and Infinite Sets? Give Examples.

Ans **Finite set** : A set is called a finite set if the process of counting the elements of that set surely comes to an end. Example is A={2,4,6,8} is a finite set because the number of elements in set A is 4.

Infinite set: A set is called an infinite set if the process of counting the number of elements in that set never ends, ie there are infinite elements in the set. Example is N= set of natural numbers .

Q4 What is the Principle of Extension?

Ans According to the Principle of Extension two sets A and B are equal if and only if they have the same members. We denote equal sets by A=B.

If A={1,3,5} and B={3,1,5}, then A=B ie A and B are equal sets.

If A={1,4,7} and B={5,4,8}, then $A \neq B$ ie A and B are unequal sets.

Q5 What is a Universal set?

Ans In any application of the theory of sets, the members of all sets under investigation usually belong to some fixed large set called the universal set. The universal set is denoted by U.

Example: In the human population studies the universal set consists of all the people in the world.

Q6 What are subsets? Give example. Write down the properties of sets.

Ans If every element in a set A is also an element of a set B, then A is called a subset of B. It can be denoted as $A \subset B$. Here B is called Superset of A.

Example: If A={1,2} and B={4,2,1} the A is the subset of B or $A \subset B$.

The properties of subsets are:-

1) Every set is a subset of itself.
2) The Null set , ie $\emptyset$ is a subset of every set.
3) If A is subset of B, and B is a subset of C then A will be the subset of C.
   If $A \subset B$ and $B \subset C \Rightarrow A \subset C$
4) A finite set having n elements has 2^n subsets.

Q7 What is a Null set?

Ans A set having no elements is called a Null set or void set. It is denoted by $\emptyset$.

Q8 Define Power Set. Write Power Set of A = {1, 2, 3}.

Ans Let B be a set then the collection of all subsets of B is called ***Power Set*** of B and is denoted by ***P(B).***

i.e. P(B) = $\{S : S \subset B\}$

If A = {1, 2, 3}

Then P(A) = {Φ, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

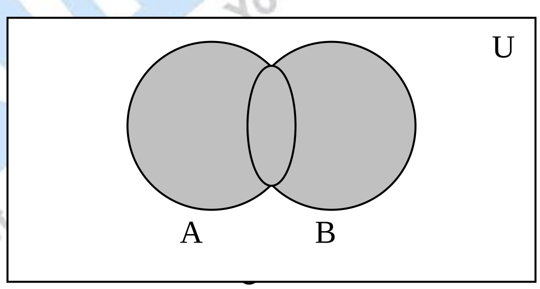
Q.9 Explain the following operations –

(a) Union (b) Intersection (c) Difference

Ans.: (a) **Union :** Let A and B be two sets then union of A and B which is denoted as $A \cup B$ is a set of elements which belongs either to A or to B or to both A and B.

So, $A \cup B = \{x : x \in A .or. x \in B\}$

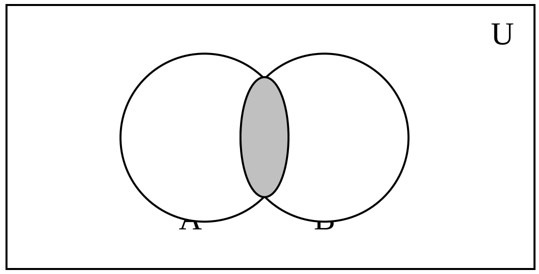
Example : If A = {1, 2, 3, 4} and B = {3, 4, 5, 6} then $A \cup B$ = {1, 2, 3, 4, 5, 6}



(b) **Intersection :** Intersection of A and B which is denoted as $A \cap B$ is a set which contains those elements that belong to both A and B.

So, $A \cap B = \{x : x \in A .and. x \in B\}$

Example : If A = {1, 2, 3, 4} and B = {3, 4, 5, 6} then $A \cap B$ = { 3, 4}



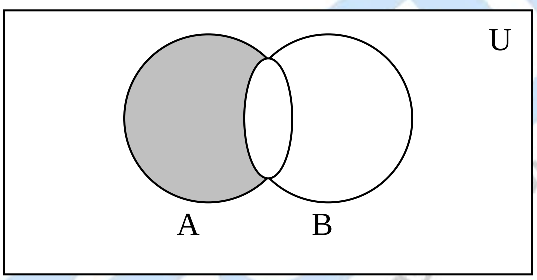
$A \cap B$

(c) **Difference :** Let A and B be two sets. The difference of A and B which is written as A - B, is a set of all those elements of A which do not belongs to B.

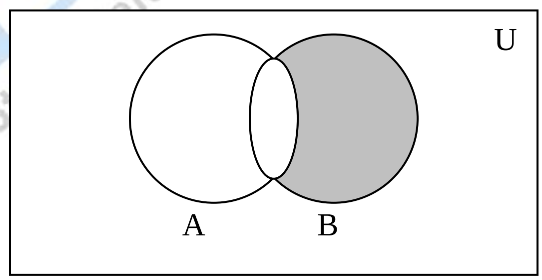
So, $A - B = \{x : x \in A.and.x \notin B\}$

Similarly, $B - A = \{x : x \in B.and.x \notin A\}$

Example : If A = {1, 2, 3, 4} and B = {3, 4, 5, 6} then A - B = { 3, 4} and B – A = {5, 6}



A – B



B - A

Q.10 Define symmetric difference of two sets If A = {2, 3, 4} and B = {3, 4, 5, 6}, Find $A \oplus B$.

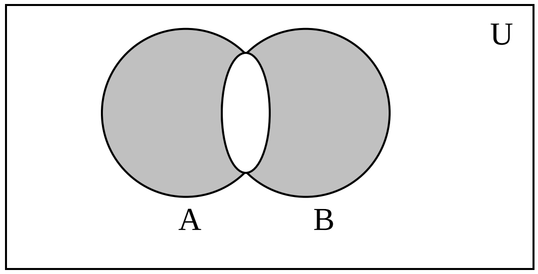
Ans.: Let A and B be two sets, the symmetric difference of A and B is the set $(A - B) \cup (B - A)$ and is denoted by $A \Delta B$ or $A \oplus B$

Thus, $A \Delta B = (A - B) \cup (B - A)$ = $\{x : x \notin A \cap B\}$

A = {2, 3, 4} and B = {3, 4, 5, 6}

A – B = {2} and B – A = {5, 6}

$$A\Delta B=(A-B)\bigcup(B-A)=\{2\}\bigcup\{5,6\}=\{2,5,6\}$$



$A\Delta B$

Q.11 State De Morgan's Law.

Ans.: If A and B are any two sets then

(i) $(A\bigcup B)'=A'\bigcap B'$ and (ii) $(A\bigcap B)'=A'\bigcup B'$

Q.12 Prove the following relation –

$n(A\bigcup B)=n(A)+n(B)-n(A\bigcap B)$

Ans.: If A and B are two sets then we know that

$$A\bigcup B=(A\bigcap B')\bigcup(A\bigcap B)\bigcup(A'\bigcap B)$$

Hence by sum rule –

$$n(A\bigcup B)=n(A\bigcap B')+n(A\bigcap B)+n(A'\bigcap B) \quad \text{\_\_\_\_\_\_\_\_\_\_(1)}$$

Again $A=(A\bigcap B')\bigcup(A\bigcap B)$

By sum rule –

$$n(A)=n(A\bigcap B')+n(A\bigcap B) \quad \text{\_\_\_\_\_\_\_\_\_\_(2)}$$

Similarly

$$n(B)=n(A\bigcap B)+n(A'\bigcap B) \quad \text{\_\_\_\_\_\_\_\_\_\_(3)}$$

Now eq^(2) + eq^(3) gives

$$n(A)+n(B)=n(A\bigcap B)+n(A\bigcap B)+n(A\bigcap B')+n(A'\bigcap B)$$

=> $n(A)+n(B)-n(A\bigcap B)=n(A\bigcap B)+n(A\bigcap B')+n(A'\bigcap B)$

From eq^(1)

$$n(A)+n(B)-n(A\bigcap B)=n(A\bigcup B)$$

Hence proved.

Q.13 What is Rusell's Paradox.

Ans For a collection to be a set it is necessary that we should be able to decide whether it belongs to the set or not. The assumption that every collection is a set leads to a paradox known as **Russel Paradox** .

Consider following example:

A barber in a certain town shaved all those who did not shave themselves and only those.

If S be the collection of all those people whom the barber shaved, is the barber the member of the set S or not ?

From the above example we get two contradictory statements:

(a) The barber is a member of set S because he shaves himself.

(b) The barber is not a member of the set because he shaves only those who did not shave themselves.

Q.14 Prove that –

A x B ≠ B x A

Ans.: A = {a, b} and B = {1, 2, 3}

A x B = { (a, 1) (a, 2) (a, 3) (b, 1) (b, 2) (b, 3)}

B x A= { (1, a) (1, b) (2, a) (2, b) (3, a) (3, b)}

Here A x B ≠ B x A

Q.15 Show that the relation 'is congruent to' on the set of all triangles in plane is an equivalence relation.

Ans.: **Proof :** Let S be the set of all triangles in a plane and R be the relation on S defined by $(\Delta_1,\Delta_2)\in R \Leftrightarrow$ triangle Δ_1 is congruent to triangle Δ_2.

(i) **Reflexivity :** for each triangle $\Delta \in S$, we have

$\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R \forall \Delta \in S$

$\Rightarrow$ R is reflexive on S.

(ii) **Symmetry :** Let Δ_1 and $\Delta_2 \in S$ such that $(\Delta_1, \Delta_2) \in R$, then

$(\Delta_1, \Delta_2) \in R \quad \Rightarrow \Delta_1 \cong \Delta_2$

$\Rightarrow \Delta_2 \cong \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\therefore$ R is symmetric on S.

(iii) **Transitivity :** Let $\Delta_1, \Delta_2, \Delta_3 \in S$ such that $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$, then

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2$

$(\Delta_2, \Delta_3) \in R \Rightarrow \Delta_2 \cong \Delta_3$

Since $\Delta_1 \cong \Delta_2$ and $\Delta_2 \cong \Delta_3 \Rightarrow \Delta_1 \cong \Delta_3 \Rightarrow (\Delta_1 \cong \Delta_3) \in R$

So, R is transitive.

Hence R is on equivalence relation.

Q.16 Let N be the set of all natural numbers and Let R be a relation on Nx N, defined by (a, b) R (c, d) $\Leftrightarrow$ ad = bc for all (a, b), (c, d) $\in$ N x N. Show that R is an equivalence relation on Nx N.

Ans.: (i) **Reflexivity :** Let (a, b) be an arbitrary element of N x N, then

$(a, b) \in NxN \Rightarrow a, b \in N$

$\Rightarrow ab = ba$

$\Rightarrow$ (a, b) R (b, a)

(by commutativity of multiplication on N)

Thus (a, b) R (b, a) for all $(a, b) \in NxN$.

So, R is reflexive.

(ii) **Symmetry :** Let (a, b), (c, d) $\in$ N x N be such that (a, b) R (c, d), then

(a, b) R (c, d) $\Rightarrow ad = bc$

$\Rightarrow cb = da$

(by commutativty of multiplication on N)

$\Rightarrow$ (c, d) R (a, b)

Thus, (a, b) R (c, d) $\Rightarrow$ (c, d) R a, b) for all (a, b), (c, d) $\in$ N x N

So, R is symmetric on N x N.

(iii) **Transitivity:** Let (a, b), (c, d), (e, f) $\in$ N x N be such that (a, b) R (c, d) and (c, d) R (e, f), then

(a, b) R (c, d) $\Rightarrow ad = bc$

and (c, d) R (e, f) $\Rightarrow cf = de$

$\Rightarrow (ad)(cf) = (bc)(de)$

$\Rightarrow af = be$

$\Rightarrow$ (a, b) R (e, f)

Thus (a, b) R (c, d) and (c, d) R (e, f) $\Rightarrow$ (a, b) R (e, f) for all (a, b), (c, d), (e, f) $\in$ N x N

So, R is transitive.

Hence, R being reflexive symmetric and transitive is an equivalence relation on N x N.

Q.17 If A = {1, 2, 3, 4, 5, 6, 7} which of the following two is a partition giving rise to an equivalence relation.

(i) **A_1 = {1, 3, 5}** **A_2 = {2}** **A_3 = {4, 7}**

(ii) **B_1 = {1, 2, 5, 7}** **B_2 = {3}** **B_3 = {4, 6}**

Ans.: (i) Since $A_1 \cup A_2 \cup A_3$ = {1, 2, 3, 4, 5, 7} $\neq$ A

So sets A_1 , A_2 , A_3 do not form a partition of A.

(ii) Since $B_1 \cup B_2 \cup B_3$ = {1, 2, 3, 4, 5, 7} =A and B_1 , B_2 , B_3 are disjoint sets.

Hence B_1 , B_2 , B_3 form a partition of A.

Q.18 If R and S are two equivalence relations on a set A, then prove that $R \cap S$ is an equivalence relation.

Ans.: It is given that R and S are equivalence relation on A. We have to show that $R \cap S$ is an equivalence relation.

(i) Reflexivity : Let $a \in A$ then

$(a, a) \in R$ and $(a, a) \in S$ [$\because$ R & S are reflexive]

$\Rightarrow (a, a) \in R \cap S$

Thus $(a, a) \in R \cap S$ for all $a \in A$, so $R \cap S$ is a reflexive relation on A.

(ii) **Symmetry :** Let $a, b \in A$ such that $(a, b) \in R \cap S$

$(a, b) \in R \cap S \Rightarrow (a, b) \in R$ and $(a, b) \in S$

$\Rightarrow (b, a) \in R$ and $(b, a) \in S$

[$\because$ R & S are symmetric]

$\Rightarrow (b, a) \in R \cap S$

Thus $(a, b) \in R \cap S \Rightarrow (b, a) \in R \cap S$

$\therefore$ $R \cap S$ is symmetric relation.

(iii) **Transitivity:** Let $a, b, c \in A$ such that $(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$(a, b) \in R \cap S$ and $(b, c) \in R \cap S$

$\Rightarrow (a, b) \in R$ and $(a, b) \in S$ and $(b, c) \in R$ and $(b, c) \in S$

$(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R$ [$\because$ R is transitive]

$(a, b) \in S$ and $(b, c) \in S \Rightarrow (a, c) \in S$ [$\because$ S is transitive]

$(a, c) \in R$ and $(a, c) \in S$

$(a, c) \in R \cap S$

Thus $(a, b) \in R \cap S$ and $(b, c) \in R \cap S \Rightarrow (a, c) \in R \cap S$

So, $R \cap S$ is transitive

Hence $R \cap S$ is an equivalence relation on A.

Chapter-3

Graph Theory

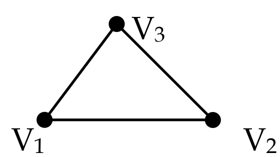
Q.1 Draw simple graphs with one, two, three and four vertices.

Ans.:

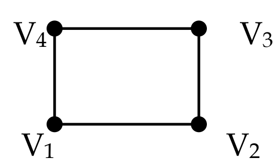
- Simple graph with one vertex

- Simple graph with two vertices

- Simple graph with three vertices



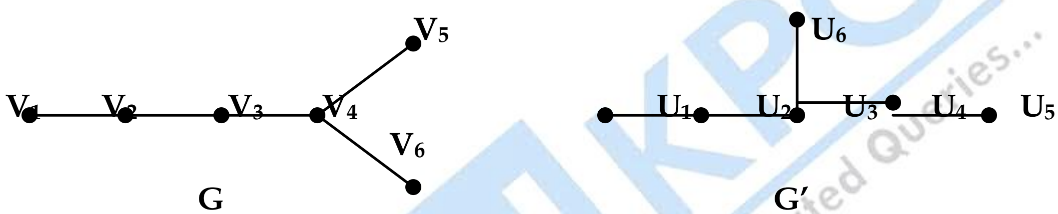
- Simple graph with four vertices



Q.2 Show that if G = (V, E) is a complete bipartite graph with n vertices then the total numbers of edges in G cannot exceed $\frac{n^2}{4}$.

Ans.: Let $K_{p,q}$ be a complete bipartite graph. The total no. of edges in $K_{p,q}$ is p.q and total no. of vertices will be (p+q). If we take $p = q = \frac{n}{2}$ then in complete bipartite graph $K_{\frac{n}{2},\frac{n}{2}}$ no. of edges will be $\frac{n}{2}.\frac{n}{2} = \frac{n^2}{4}$ which is maximum (If two numbers are equal then their product is maximum). Hence in a complete bipartite graph of n vertices the no. of edges cannot exceed $\frac{n^2}{4}$.

Q.3 Show that following two graphs are not isomorphic.



Ans.: In graph G and G′ we find that

(i) No. of vertices in G = No. of vertices in G′ = 6.

(ii) No. of edges in G = No. of edges in G′ = 5.

(iii) No. of vertices of degree one in G and G′ = 3.

No. of vertices of degree two in G and G′ = 2

No. of vertices of degree three in G and G′ = 1

i.e. Number of vertices of equal degree are equal. Although it satisfies all the three conditions but then also G and G′ are not isomorphic because corresponding to vertex V_4 in G there should be a vertex U_3 because in both G and G′ there is only one vertex of degree three. But two pendent vertices V_5 and V_6 are incident on the vertex V_4 in G whereas only one pendent vertex U_6 is incident on the vertex U_3 in G′.

Hence G and G′ are not isomorphic.

Q.4 Define the followings :-

(i) Walk (ii) Trail (iii) Path (iv) Circuit (v) Cycle

Ans.: (i) **Walk :** An alternating sequence of vertices and edges is called a ***Walk.*** It is denoted by ***'W'.***

Example :

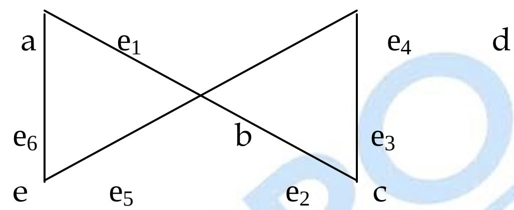


Figure (1)

Here W = ae_1 b e_2 c e_3 d is a walk.

Walk is of two types :-

(a) **Open Walk :** If the end vertices of a walk are different then such a walk is called ***Open Walk.***

Example from fig.(1) : W = a e_1 b e_2 c e_3 d is an open walk.

(b) **Closed Walk :** If a walk starts and end with same vertex then such a walk is called closed walk.

Example from fig.(1) : W = a e_6 e e_5 b e_1 a is a closed walk as it starts and end with same vertex a.

(ii) **Trail :** An open walk in a graph G in which no edge is repeated is called a ***Trail.***

Example from fig.(1) : W = a e_1 b e_2 c e_3 d is a trail.

(iii) **Path :** An open walk in which no vertex is repeated except the initial and terminal vertex is called a ***Path.***

Example for fig.(1) : W = a e_1 b e_4 d e_3 c is a path.

(iv) **Circuit :** A closed trail is called a ***Circuit.***

Example for fig.(1) : W = a e_1 b e_5 e e_6 a is a circuit.

(v) **Cycle :** A closed path is called a ***Cycle.***

Example for fig.(1) : W = a e_1 b e_5 e e_6 a is a cycle.

Q.5 State and prove Handshaking Theorem.

Ans.: **Handshaking Theorem :** The sum of degrees of all the vertices in a graph G is equal to twice the number of edges in the graph.

Mathematically it can be stated as :

$$\sum_{v \in V} \deg(v) = 2e$$

Proof : Let G = (V, E) be a graph where V = {v_1, v_2, ...} be the set of vertices and E = {e_1, e_2, ...} be the set of edges. We know that every edge lies between two vertices so it provides degree one to each vertex. Hence each edge contributes degree two for the graph. So sum of degrees of all vertices is equal to twice the number of edges in G.

Hence $\sum_{v \in V} \deg(v) = 2e$

Q.6 Explain Matrix Representation of Graphs.

Ans.: Although a pictorial representation of a graph is very convenient for a visual study, other representations are better for computer processing. A matrix is convenient and useful way of the representation of a graph to a computer for a graph. There are different types of matrices :

(i) Incidence Matrix

(ii) Circuit Matrix

(iii) Adjacency Matrix

(iv) Path Matrix etc.

Q.7 How many edges are there with 7 vertices each of degree 4?

Ans.: In graph G, there are 7 vertices and degree of each vertex is 4. So sum of the degrees of all the vertices of graph G = 7 x 4 = 28.

According to Handshaking Theorem –

$$\sum_{v \in V} \deg(v) = 2e$$

⇨ $28 = 2e$

⇨ $\boxed{e = 14}$

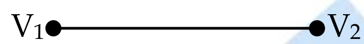
So, total no. of edges in G = 14.

Q.8 Define Regular and Complete Graph.

Ans.: **Regular Graph :** A simple graph G = (V, E) is called a ***Regular Graph*** if degree of each of its vertices are equal.

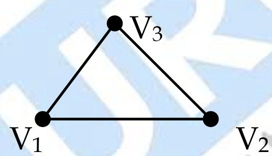
Examples :

1-



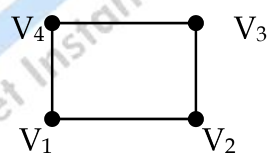
Here degree of each vertex is one. So it is regular graph.

2-



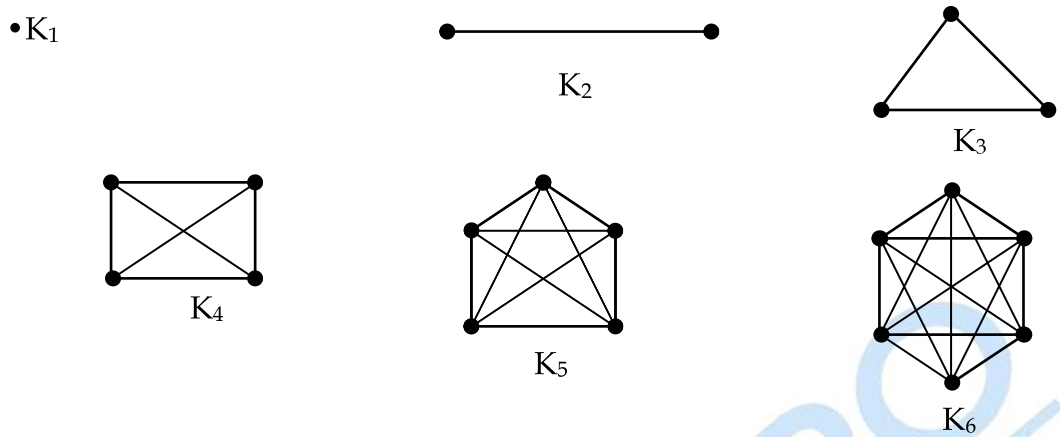
Degree of each vertex is two.

3-



Degree of each vertex is two.

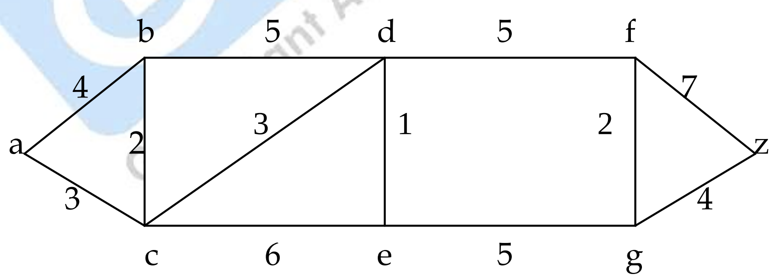
Complete Graph : A simple graph G = (V, E) is called a ***Complete Graph*** if there is exactly one edge between every pair of distinct vertices. A complete graph with n-vertices is denoted by K_n.



In a complete graph K_n total no. of edges $= \frac{n(n-1)}{2}$

i.e. size of $K_n = \frac{n(n-1)}{2}$

Q.9 Find the shortest path between the vertex a and z in the following graph.



Ans.: First we label the vertex a by permanent label 0 and rest by '∞'.

a	b	c	d	e	f	g	h
0 √	∞	∞	∞	∞	∞	∞	∞
0	4	3 √	∞	∞	∞	∞	∞
0	4 √	3	6	9	∞	∞	∞
0	4	3	6 √	9	∞	∞	∞
0	4	3	6	7 √	11	∞	∞
0	4	3	6	7	11 √	12	∞
0	4	3	6	7	11	12 √	18
0	4	3	6	7	11	12	16 √

Hence shortest path is a → c → d → e → g → z = 16

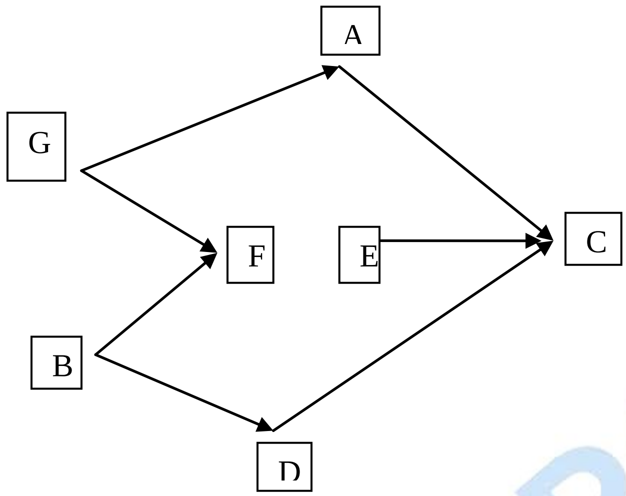
Q 10 Explain the Topological Sort.

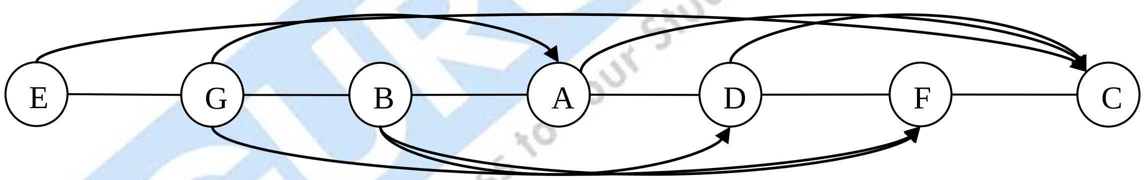
Ans Let S be a directed graph such that:

1) each vertex v_i of S represents a task, and
2) each (directed) edge (u,v) of S means that the task u must be completed before beginning the task v . Suppose such a graph S contains a cycle, say P=(u,v,w,u)

This means we must complete task u before beginning v, we must complete task v before beginning w, and we must complete task w before beginning task u. Thus we cannot begin any of the three tasks in the cycle. Accordingly, such a graph S representing tasks and prerequisite relation cannot have any cycles or we can say such a graph S must be cycle free or acyclic. A directed acyclic (cycle free) graph is called a dag for short.

A fundamental operation on a dag S is to process the vertices one after the other so that the vertex u is always processed before vertex v whenever (u,v) is an edge. Such a linear ordering T of the vertices of S, which may not be unique, is called a topological sort.





Above is a Topological sort

Chapter-4

Group

Q1 What is a group?

Ans Let G be a non void set with a binary operation * that assigns to each ordered pair (a,b) of elements of G an element of G denoted by a * b. We say that G is a group under the binary operation *, if the following three properties are satisfied:

1) **Associativity**: the binary operation * is associative i.e.

   $a*(b*c)=(a*b)*c \quad , \quad \forall\, a,b,c \in G$

2) **Identity**: there is an element e, called the identity , in G, such that

   $a*e=e*a=a \quad , \quad \forall\, a \in G$

3) **Inverse:** For each element a in G, there is an element b in G, called an inverse of a, such that

   $a*b=b*a=e \ , \ \forall\, a , b \in G$

   Note: If a group has the property that

   $a*b=b*a$, i.e commutative law holds then the group is called an abelian.

Q2 What are the elementary properties of Groups ?

Ans The elementary properties of Groups can be understood by the following theorems:-

Theorum1):-

(i) Statement :- In a Group G, there is only one identity element(uniqueness of identity)

Proof:- let e and e′ are two identities in G and let $a \in G$

$\therefore ae = a \rightarrow$ (i)

$\therefore ae' = a \rightarrow$ (ii)

R.H.S of (i) and (ii) are equal

$\Rightarrow$ ae=ae′

Thus by the left cancellation law, we obtain

e=e′

$\Rightarrow$ there is only one identity element in G for any a $\in$ G. Hence the theorem is proved.

(ii) Statement:- For each element a in a group G, there is a unique element b in G such that ab= ba=e (uniqueness if inverses)

Proof:- let b and c are both inverses of a $\in$ G

Then ab = e and ac = e

$\because$ c = ce { existence of identity element }

$\Rightarrow$ c = c (ab) { $\because$ ab = e }

$\Rightarrow$ c = (c a) b

$\Rightarrow$ c = (ac) b { $\because$ ac = ca }

$\Rightarrow$ c = eb

$\Rightarrow$ c = b { $\because$ b = eb }

Hence inverse of a $\in$ G is unique.

Theorem 2):-

(i) Statement:- In a Group G , $(a^{-1})^{-1} = a$, $\forall$ a $\in$ G

Proof:- We have $a a^{-1} = a^{-1} a = e$

Where e is the identity element of G

Thus a is inverse of $a^{-1} \in G$

i.e. $(a^{-1})^{-1} = a$, $\forall$ a $\in$ G

(ii) Statement:- In a Group G, $(a b)^{-1} = b^{-1} a^{-1}$, $\forall$ a , b $\in$ G

Proof:- By associatively we have

$(b^{-1} a^{-1}) ab = b^{-1} (a^{-1}a)b$

$\Rightarrow$ $(b^{-1} a^{-1})\, ab = b^{-1} (e) b$ $\{\because a^{-1}a = e\}$

$\Rightarrow$ $(b^{-1} a^{-1})\, ab = b^{-1} b$ $\{\because eb = b\}$

$\Rightarrow$ $(b^{-1} a^{-1})\, ab = e$, $\{\because b^{-1} b = e\}$

Similarly

$(ab)(b^{-1} a^{-1}) = a\,(bb^{-1})\, a^{-1}$

$\Rightarrow (ab)(b^{-1} a^{-1}) = a\,(e)\, a^{-1}$

$\Rightarrow (ab)(b^{-1} a^{-1}) = a\, a^{-1}$

$\Rightarrow (ab)(b^{-1} a^{-1}) = e$ $\{\because aa^{-1} = e\}$

Thus $(b^{-1} a^{-1})\, ab = (ab)(b^{-1} a^{-1}) = e$

$\therefore$ $b^{-1} a^{-1}$ is the inverse of ab

i.e. $b^{-1} a^{-1} = (ab)^{-1}$

Hence the theorem is proved.

Theorum3):-

In a group G, the left and right cancellation laws hold i.e.

(i) ab = ac implies b=c
(ii) ba=ca implies b=c

Proof

(i) Let ab=ac

Premultiplying a^{-1} on both sides we get

$a^{-1}(ab) = a^{-1}(ac)$

$\Rightarrow (a^{-1} a)\, b = (a^{-1} a)\, c$

$\Rightarrow eb = ec$

$\Rightarrow b = c$

Hence Proved

Proof

(ii) Let ba=ca

Post multiplying a^{-1} on both sides

$\Rightarrow (ba)\, a^{-1} = (ca)\, a^{-1}$

$\Rightarrow b\,(aa^{-1}) = c(aa^{-1})$

$\Rightarrow be=ce$

$\Rightarrow b=c$

Hence the theorem is proved

Q3 What is the order of an element in the context of Groups ?

Ans The order of an element x in a group G is the smallest positive integer n such that $x^n = e$.

The order of an element x is denoted by O(x) or $|x|$. If no such integer exists we say x has infinite order. So to find the order of an element $x \in G$, we need to compute the sequence of products $x, x^2, x^3, \ldots$..until we reach the identity for the first time.The exponent of this product is the order of x.

Q4 Define Subgroup.

Ans If a nonvoid subset H of a group G is itself a group under the operation of G, we say H is a subgroup of G.

Theorum:- A subset H of a group G is a subgroup of G if:

(i) the identity element $e \in H$.

(ii) H is closed under the operation of G i.e. if $a,b \in H$, then $ab \in H$ and

(ii) H is closed under inverses, that is if $a \in H$ then $a^{-1} \in H$.

Q5 What are Cyclic subgroups and Cyclic Groups ?

Ans <u>**Cyclic Subgroup**</u>:- A Subgroup K of a group G is said to be **cyclic subgroup** if there exists an element $x \in G$ such that every element of K can be written in the form x^n for some $n \in Z$.

The element x is called generator of K and we write $K=<x>$

<u>**Cyclic Group**</u>:- In the case when $G=<x>$, we say G is cyclic and x is a generator of G. That is, a group G is said to be cyclic if there is an element $x \in G$ such that every element of G can be written in the form x^n for the some $n \in Z$.

Example, the group G= { 1, -1, i,-i} under usual multiplication is a finite cyclic group with i as generator , since $i^1 = i$, $i^2 = -1$, $i^3 = -i$ and $i^4 = 1$

Q6 Explain Permutation Groups.

Ans A one- to –one mapping σ of the set {1,2,3,…n} onto itself is called a permutation. Let A be a nonvoid set, and let S_A be the collection of all permutations of A. Thus permutation group of set A denoted by S_A is a set of permutations of A that forms a group under function composition.

Let A={1,2,3…n}, the set of all permutations of A is called the symmetric group of degree n and is denoted by S_n.

Elements of S_n may be written as $\sigma = \begin{pmatrix} 1 & 2 & 3 \ldots & n \\ \sigma(1) & \sigma(2) & \sigma(3) \ldots\ldots & \sigma(n) \end{pmatrix}$

The symmetric group S_n has n! elements

Q 7 What are Cosets ?Explain it.

Ans Let H be a subgroup of a group G. A left coset of H in G is a subset of G whose elements may be expressed as

$xH=\{ xh \mid h \in H \}$

for any $x \in G$. The element x is called a representation of the coset .

Similarly, a right coset of H in G is a subset that may be expressed as

$Hx= \{ hx \mid h \in H \}$, for any $x \in G$

Thus complexes xH and Hx are called respectively a left coset and a right coset.

If the group operation is additive (+) then a left coset is denoted as

$x + H=\{ x+h \mid h \in H \}$

and a right coset is denoted by

$H + x = \{ h+x \mid h \in H \}$

Q8 What are Normal Subgroups ?

Ans Let G be a group. A subgroup H of G is said to be a normal subgroup of G if for all $h \in H$ and $x \in G$, $x h x^{-1} \in H$.

If $x H x^{-1} = \{ x h x^{-1} \mid h \in H \}$ then H is normal in G if and only if $xHx^{-1} \subseteq H$, $\forall$ $x \in G$

We hereby discuss a theorem:

Statement: If G is an abelian group, then every subgroup H of G is normal in G.

Proof: Let any $h \in H$, $x \in G$, then

$x h x^{-1} = x (h x^{-1})$

$x h x^{-1} = (x x^{-1}) h$

$x h x^{-1} = e h$

$x h x^{-1} = h \in H$

Hence H is normal subgroup of G.

Q 9 Define Group Homomorphism.

Ans A homomorphism is a mapping $f: G \to G'$ such that $f(xy)=f(x) f(y)$, $\forall$ $x,y \in G$ The mapping f preserves the group operation although the binary operations of the group G and G′ are different. Above condition is called the homomorphism condition.

Kernel of Homomorphism:- The Kernel of a homomorphism f from a group G to a group G′ with identity e′ is the set $\{ x \in G \mid f(x) = e' \}$

The kernel of f is denoted by Ker f.

If $f: G \to G'$ is a homomorphism of G into G′, then the image set of f is the range, denoted by f(G), of the map f. Thus

$Im(f) = f(G)= \{ f(x) \in G' \mid x \in G \}$

If $f(G) = G'$, then G′ is called a homomorphic image of G.

Note:- A group homomorphism $f: G \to G'$ which is bijective is known as an isomorphism.

Q 10 The mapping $f: R \to R^+$ defined by $f(x) = e^x$ is an isomorphism from the group (R,+) to the group $(R^+ , .)$

Ans It is clear that the mapping f is bijective in nature and satisfies homomorphism condition.

$f(x+y) = e^{x+y} = e^x \cdot e^y = f(x)\, f(y), \ \forall\, x, y \in R$

Hence f is an isomorphism.

Chapter-5

Finite State Machines and Languages

Q1 What are alphabets ?

Ans The finite non-void set A of symbols is called alphabet or vocabulary. Symbols can be 0,1 or a,b,c,….,z

(i) A={0,1} is a binary alphabet.

(ii) A={ a,b,c,….., z}

Q2 What are word or sentence ?

Ans If we select some symbols out of the alphabet A to form a finite sequence or a finite string, then what we obtain is called a word or sentence.

Example: string w=101011 is a sentence formed out of the element of alphabet A={0,1}

Similarly, w=aabcb is a sentence formed out of the alphabet A={a,b,c}

Q3 Define Empty word or empty string.

Ans Any string is called empty string, if it is formed by none of the symbols/letters of alphabet A. Empty string is denoted by λ

Length of any string w is denoted by |w| if w=101011 then |w|=6.

Q4 Define Powers of an alphabet.

Ans Let A is an alphabet. Then all the possible strings(words) formed from the symbols/letters of an alphabet A of each having length k are denoted by A^k and is called power of an alphabet A.

Example:- for any alphabet A , A^0= { λ }.

If A={0,1} , A^1 = {0,1}, A^2 = { 00,01,10,11}, A^3 = {000,001,010,011,100,101,110,111 }

A^* is a set consisting of all the words formed from the letters of alphabet A.

If A={0,1} then

$A^* = \{\lambda, 0,1,00,01,10,11,000,001,\ldots\}$

Therefore, $A^* = A^0 \cup A^1 \cup A^2 \cup \ldots$

Q5 What is Concatenation of words ?

Ans Let u and v are two words on alphabet A. Then uv is called the concatenation of words u and v, obtained by writing u and v one after the other.

Let u = 10011 and v=011011

Therefore uv=10011011011

vu=01101110011

clearly, for any three words u,v,w

(uv)w=u(vw)

Q6 What are subwords and initial segments?

Ans let $u=a_1a_2\ldots a_n$ is any word on alphabet A. Then string $w= a_j a_{j+1} \ldots a_k$ is called a subword of word u.

When j=1 i.e. $w=a_1a_2\ldots a_k$ is called the initial segment of u.

Q7 What is a Language. Explain.

Ans The collection of words on an alphabet A is called a language. If A is an alphabet and

$L \subseteq A^*$ where $A^* = \{\lambda, 0,1,00,01,10,11,\ldots\}$, then L is called language.

If L is a language on alphabet A, then L will be also a language on any superset of A.

Example: Let A={a,b}

$L_1 = \{a, ab, ab^2, \ldots\}$, $L_2 = \{a^2, a^4, a^6, \ldots\}$ are languages on alphabet A.

Note:-

(1) A^* is language on alphabet A.
(2) φ i.e. the empty language is also a language on alphabet A.
(3) $\{\lambda\}$, where λ is empty word is also a language.

Q 8 What is the Finite State Automata ?

Ans A finite state automata (FSA) denoted by M consists of five parts:-

1) A finite set (alphabets) A of inputs.
2) A finite set S of (internal) states .
3) A subset Y of S (whose elements are called accepting or "yes" states).
4) An initial state s_0 in S.
5) A next – state function F from S x A into S.

Such an automation M is denoted by M=(A,S,Y,s_0,F) when we want to indicate its five parts.

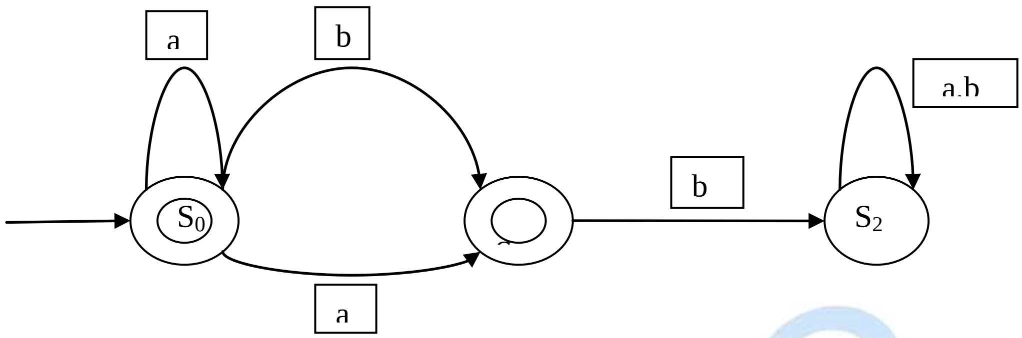
Example:- The following defines an automation M with two input symbols and three states:

1) A={a,b}, input symbols.
2) S={s_0,s_1,s_2}, internal states.
3) Y={ s_0,s_1 }, "yes" states.
4) s_0 , initial state.
5) Next-state function F: S x A → S defined by

   F(s_0 , a)= s_0 , F(s_1 , a)= s_0 , F(s_2 , a)= s_2

   F(s_0 , b)= s_1 , F(s_1 , b)= s_2 , F(s_2 , b)= s_2

F	a	b
s_0	s_0	s_1
s_1	s_0	s_2
s_2	s_2	s_2



Q 9 What is a Phase Structure Grammar or simply a Grammar ?

Ans A Phase structure grammer consists of four parts:

1) A finite set (vocabulary) V.
2) A subset T of V whose elements are called terminals.
3) A non-terminal symbol S called the start symbol.
4) A finite set P of productions. A production is an ordered pair(a,b) usually written as $a \rightarrow b$ where a and b are words in V. Each production in P must contain at least one nonterminal on its left side.

Such a grammer G is denoted by G=G(V,T,S,P) when we want to indicate its four parts.
Terminals are denoted by lower case a,b,c.. and nonterminals will be denoted by capital A,B,C..with S as the start symbol.

Q10 What is the concept of Finite State Machines ?

Ans A finite state machine (FSM) is similar to a finite state automation (FSA) except that the finite state machine "prints" an output using an output alphabet distinct from the input alphabet. The formal definition follows.
A Finite state machine M consists of six parts:

1) A finite set A of input symbols.
2) A finite set S of "internal" states.
3) A finite set Z of output symbols.
4) An initial state s_0 in S.
5) A next-state function f from S x A into S.

6) An output function g from S x A into z.

Such a machine M is denoted by M=M(A,S,Z,s_0 , f , g) when we want to indicate its six parts.

Chapter-6

Posets and Lattices

Q1 What is a Partially ordered set or Poset ?

Ans Let R is a relation on a set S satisfying the following three properties:

1) Reflexive: For any a ∈ S, we have a R a.
2) Antisymmetric: If a R b and b R a, then a=b.
3) Transitive: If a R b and b R c, then a R c.

Then R is called a partial order.
The set S with the partial order is called a partial ordered set or simply a poset.

Q2 What are Supremum and Infimum ?

Ans Let A be a subset of a partially ordered set S. An element M in S is called an upper bound of A if M succeeds every element of A, i.e. if for every x in A, we have

x <=M

If an upper bound of A precedes every other upper bound of A, then it is called the supremum of A and is denoted by

Sup(A)

An element m in a poset S is called a lower bound of a subset A of S if m precedes every element of A, i.e. if ,for every y in A, we have

m <=y

If a lower bound of A succeeds every other lower bound of A, then it is called the infimum of A and is denoted by

Inf(A)

Q 3 Explain Well-Ordered sets.

Ans An ordered set S is said to be well – ordered if every subset of S has a first element
One of the example of a well ordered set is the set N of positive integers.

A well ordered set is linearly ordered. For if a,b $\in$ S, then { a, b} has a first element; hence a and b are comparable. Every subset of a well –ordered set is well-ordered.

Q4 What are Lattices ?

Ans Let L be a non empty set closed under two binary operations called meet and join, denoted by $\wedge$ and $\vee$. Then L is called a lattice if the following axioms hold where a,b,c are elements in L :

1) Commutative law:-
   (a) $a \wedge b = b \wedge a$ (b) $a \vee b = b \vee a$
2) Associative Law :-
   (a) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$ (b) $(a \vee b) \vee c = a \vee (b \vee c)$
3) Absorption Law:-
   (a) $a \wedge (a \vee b) = a$ (b) $a \vee (a \wedge b) = a$

Q5 What is the Duality ?

Ans The dual of any statement in a lattice (L , $\wedge$, $\vee$) is defined to be a statement that is obtained by interchanging $\wedge$ an $\vee$. For example, the dual of $a \wedge (b \vee a) = a \vee a$ is

$a \vee (b \wedge a) = a \wedge a$

Q6 Define Bounded Lattices.

Ans A lattice L is said to have a lower bound 0 if for any element x in L we have 0 <=x. L is thus said to have an upper bound I if for any x in L , we have x<=I. We say L is bounded if L has both a lower bound 0 and an upper bound I. In such a lattice we have the identities:

$a \vee I = I$, $a \wedge I = a$, $a \vee 0 = a$, $a \wedge 0 = 0$ for any element a in L

Q 7 Define Complements and complemented lattices.

Ans Let L be a bounded lattice with lower bound o and upper bound I. Let a be an element if L. An element x in L is called a complement of a if

$a \vee x = I$ and $a \wedge x = 0$

A lattice L is said to be complemented if L is bounded and every element in L has a complement.

Chapter-7

Combinatorics

Q1 What are Permutation and Combinations. Explain with examples.

Ans **Permutation** : Permutation means *arrangement* of things. The word *arrangement* is used, if the order of things *is considered*.

Number of permutations of 'n' different things taken 'r' at a time is given by:-

$$^{n}P_{r} = n!/(n-r)!$$

Combination: Combination means *selection* of things. The word *selection* is used, when the order of things has *no importance*.

Example: Suppose we have to form a number of consisting of three digits using the digits **1,2,3,4**, To form this number the digits have to be *arranged*. Different numbers will get formed depending upon the order in which we arrange the digits. This is an example of *Permutation*.

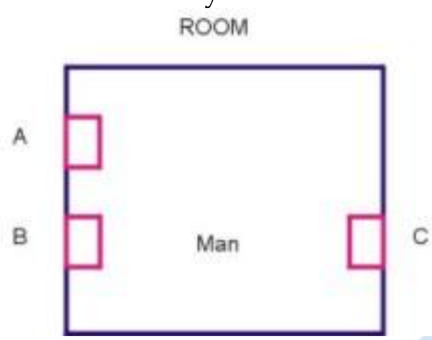
Now suppose that we have to make a team of 11 players out of 20 players, This is an example of *combination*, because the order of players in the team will not result in a change in the team. No matter in which order we list out the players the team will remain the same! For a different team to be formed at least one player will have to be changed.

Q2 Explain the fundamental principles of counting.

Ans The two fundamental principles of counting:

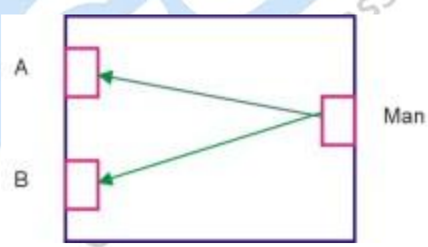
Addition rule : If an experiment can be performed in 'n' ways, & another experiment can be performed in 'm' ways then either of the two experiments can be performed in (m+n) ways. This rule can be extended to any finite number of experiments.

Example: Suppose there are 3 doors in a room, 2 on one side and 1 on other side. A man want to go out from the room. Obviously he has '3' options for it. He can come out by door 'A' or door 'B' or door 'C'.



Multiplication Rule : If a work can be done in m ways, another work can be done in 'n' ways, then both of the operations can be performed in m x n ways. It can be extended to any finite number of operations.

Example.: Suppose a man wants to cross-out a room, which has 2 doors on one side and 1 door on other site. He has 2 x 1 = 2 ways for it.



Q3 What is the Factorial of any number?

Ans Factorial n : The product of first 'n' natural numbers is denoted by n!.

n! = n(n-1) (n-2) ………………..3.2.1.

Ex. 5! = 5 x 4 x 3 x 2 x 1 =120

Note 0! = 1

Proof n! =n, (n-1)!

Or (n-1)! = [n x (n-1)!]/n = n! /n

Putting n = 1, we have

O! = 1!/1

or 0 = 1

Q4 Explain the various types of permutations.

Ans (1) Number of permutations of 'n' different things taken 'r' at a time is given by:-

^nP_r = n!/(n-r)!

Proof: Say we have 'n' different things $a_1, a_2, \ldots, a_n$.

Clearly the first place can be filled up in 'n' ways. Number of things left after filling-up the first place = n-1

So the second-place can be filled-up in (n-1) ways. Now number of things left after filling-up the first and second places = n - 2

Now the third place can be filled-up in (n-2) ways.

Thus number of ways of filling-up first-place = n

Number of ways of filling-up second-place = n-1

Number of ways of filling-up third-place = n-2

Number of ways of filling-up r-th place = n – (r-1) = n-r+1

By multiplication – rule of counting, total no. of ways of filling up, first, second -- rth-place together :-

n (n-1) (n-2) ------------ (n-r+1)

Hence:

$^{n}P_{r}$ = n (n-1)(n-2) --------------(n-r+1)

= [n(n-1)(n-2)----------(n-r+1)] [(n-r)(n-r-1)-----3.2.1.] / [(n-r)(n-r-1)] ----3.2.1

$^{n}P_{r} = n!/(n-r)!$

3) Number of permutations of 'n' different things taken all at a time is given by:-

$^{n}P_{n}$ = n!

Proof :

Now we have 'n' objects, and n-places.

Number of ways of filling-up first-place = n

Number of ways of filling-up second-place = n-1

Number of ways of filling-up third-place = n-2

Number of ways of filling-up r-th place, i.e. last place = 1

Number of ways of filling-up first, second, --- n th place = n (n-1) (n-2) ------ 2.1.

$^{n}P_{n}$ = n!

Concept.

We have ^nP_r = n!/n-r

Putting r = n, we have :-

^nP_r = n! / (n-r)

But ^nP_n = n!

Clearly it is possible, only when n! = 1

Hence it is proof that 0! = 1

Note : Factorial of negative-number is not defined. The expression –3! has no meaning.

4) Number of permutations of n-thing, taken all at a time, in which 'P' are of one type, 'g' of them are of second-type, 'r' of them are of third-type, and rest are all different is given by :-

n!/p! x q! x r!

4) Number of permutations of n-things, taken 'r' at a time when each thing can be repeated r-times is given by = n^r.

Proof.

Number of ways of filling-up first –place = n

Since repetition is allowed, so

Number of ways of filling-up second-place = n

Number of ways of filling-up third-place

Number of ways of filling-up r-th place = n

Hence total number of ways in which first, second ----r th, places can be filled-up

= n x n x n ------------- r factors.

= $\mathbf{n^r}$

Q5). How many different signals can be made by 5 flags from 8-flags of different colours?

Ans. Number of ways taking 5 flags out of 8-flage = $^8P_5$

= 8!/(8-5)!

= 8 x 7 x 6 x 5 x 4 = 6720

Q6 How many words can be made by using the letters of the word "SIMPLETON" taken all at a time?

Ans. There are '9' different letters of the word "SIMPLETON"

Number of Permutations taking all the letters at a time = $^9P_9$

Q7 In how many ways can the letters of the word "Pre-University" be arranged?

Ans The possible ways are=13!/2! X 2! X 2!

Q 8 A child has 3 pocket and 4 coins. In how many ways can he put the coins in his pocket.

Ans. First coin can be put in 3 ways, similarly second, third and forth coins also can be put in 3 ways.

So total number of ways = 3 x 3 x 3 x 3 = 3^4 = 81

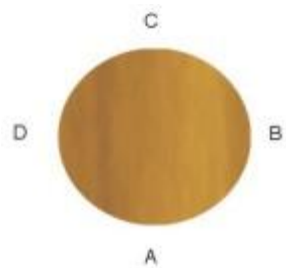
Q9 What are circular permutations? Explain

Ans There are two cases of circular-permutations:-

(a) If clockwise and anti clock-wise orders are different, then total number of circular-permutations is given by (n-1)!

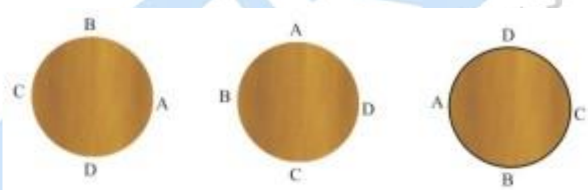
(b) If clock-wise and anti-clock-wise orders are taken as not different, then total number of circular-permutations is given by (n-1)!/2!

Proof(a):



(a) Let's consider that 4 persons A,B,C, and D are sitting around a round table

Shifting A, B, C, D, one position in anticlock-wise direction, we get the following agreements:-



Thus, we use that if 4 persons are sitting at a round table, then they can be shifted four times, but these four arrangements will be the same, because the sequence of A, B, C, D, is same. But if A, B, C, D, are sitting in a row, and they are shifted, then the four linear-arrangement will be different.

Hence if we have '4' things, then for each circular-arrangement number of linear-arrangements =4

Similarly, if we have 'n' things, then for each circular – agreement, number of linear – arrangement = n.

Let the total circular arrangement = p

Total number of linear–arrangements = n.p

Total number of linear–arrangements = n. (number of circular-arrangements)

Or Number of circular-arrangements = 1 (number of linear arrangements)

n = 1(n!)/n

circular permutation = (n-1)!

Proof

(b) When clock-wise and anti-clock wise arrangements are not different, then observation can be made from both sides, and this will be the same. Here two permutations will be counted as one. So total permutations will be half, hence in this case.

Circular–permutations = (n-1)!/2

Note: Number of circular-permutations of 'n' different things taken 'r' at a time:-

(a) If clock-wise and anti-clockwise orders are taken as different, then total number of circular-permutations = ${}^nP_r/r$

(b) If clock-wise and anti-clockwise orders are taken as not different, then total number of circular – permutation = ${}^nP_r/2r$

Q10 How many necklace of 12 beads each can be made from 18 beads of different colours?

Ans. Here clock-wise and anti-clockwise arrangement s are same.

Hence total number of circular–permutations: $^{18}P_{12}/2x12$

$= 18!/(6 \quad x \quad 24)$

Q11 What are restricted permutations ?

Ans (a) Number of permutations of 'n' things, taken 'r' at a time, when a particular thing is to be always included in each arrangement

$= r\ ^{n-1}P_{r-1}$

(b) Number of permutations of 'n' things, taken 'r' at a time, when a particular thing is fixed: $= {}^{n-1}P_{r-1}$

(c) Number of permutations of 'n' things, taken 'r' at a time, when a particular thing is never taken: $= {}^{n-1}P_{r.}$

(d) Number of permutations of 'n' things, taken 'r' at a time, when 'm' specified things always come together = m! x (n-m+1) !

(e) Number of permutations of 'n' things, taken all at a time, when 'm' specified things always come together = n ! - [m! x (n-m+1)!]

Example: How many words can be formed with the letters of the word 'OMEGA' when:

(i) 'O' and 'A' occupying end places.

(ii) 'E' being always in the middle

(iii) Vowels occupying odd-places

(iv) Vowels being never together.

Ans.

(i) When 'O' and 'A' occupying end-places

=> M.E.G. (OA)

Here (OA) are fixed, hence M, E, G can be arranged in 3! ways

But (O,A) can be arranged themselves is 2! ways.

=> Total number of words = 3! x 2! = 12 ways.

(ii) When 'E' is fixed in the middle

=> O.M.(E), G.A.

Hence four-letter O.M.G.A. can be arranged in 4! i.e 24 ways.

(iii) Three vowels (O,E,A,) can be arranged in the odd-places (1^{st}, 3^{rd} and 5^{th}) = 3! ways.

And two consonants (M,G,) can be arranged in the even-place (2^{nd}, 4^{th}) = 2 ! ways

=> Total number of ways= 3! x 2! = 12 ways.

(iv) Total number of words = 5! = 120!

If all the vowels come together, then we have: (O.E.A.), M,G

These can be arranged in 3! ways.

But (O,E.A.) can be arranged themselves in 3! ways.

=> Number of ways, when vowels come-together = 3! x 3!

= 36 ways

=> Number of ways, when vowels being never-together

= 120-36 = 84 ways.

Q12 Give the formula for calculating combinations.Explain

Ans Number of Combination of 'n' different things, taken 'r' at a time is given by:-

$^nC_r = n! / r! \times (n-r)!$

Proof: Each combination consists of 'r' different things, which can be arranged among themselves in r! ways.

=> For one combination of 'r' different things, number of arrangements = r!

For ^nC_r combination number of arrangements: r ^nC_r

=> Total number of permutations = r! ^nC_r --------------- (1)

But number of permutation of 'n' different things, taken 'r' at a time

= ^nP_r -------(2)

From (1) and (2) :

$$^nP_r = r! \,.\, ^nC_r$$

or $n!/(n-r)! = r! \,.\, ^nC_r$

or $^nC_r = n!/r!x(n-r)!$

Note: ${}^{n}C_{r}$ = ${}^{n}C_{n-r}$

or ${}^{n}C_{r}$ = n!/r!x(n-r)! and ${}^{n}C_{n-r}$ = n!/(n-r)!x(n-(n-r))!

= n!/(n-r)!xr!

Q13 Explain restricted combinations.

(a) Number of combinations of 'n' different things taken 'r' at a time, when 'p' particular things are always included = ${}^{n-p}C_{r-p}$.

(b) Number of combination of 'n' different things, taken 'r' at a time, when 'p' particular things are always to be excluded = ${}^{n-p}C_{r}$

Example: In how many ways can a cricket-eleven be chosen out of 15 players? if

(i) A particular player is always chosen,

(ii) A particular is never chosen.

Ans:

(i) A particular player is always chosen, it means that 10 players are selected out of the remaining 14 players.

= Required number of ways = ${}^{14}C_{10}$ = ${}^{14}C_{4}$

= 14!/4!x19! = 1365

(ii) A particular players is never chosen, it means that 11 players are selected out of 14 players.

=> Required number of ways = ${}^{14}C_{11}$

= $14!/11!\times 3!$ = 364

(iii) Number of ways of selecting zero or more things from ‘n’ different things is given by:- 2^n-1

Proof: Number of ways of selecting one thing, out of n-things = $^nC_1$

Number of selecting two things, out of n-things = $^nC_2$

Number of ways of selecting three things, out of n-things = $^nC_3$

Number of ways of selecting ‘n’ things out of ‘n’ things = ^nC_n

=>Total number of ways of selecting one or more things out of n different things

= $^nC_1 + {}^nC_2 + {}^nC_3$ + ------------- + ^nC_n

= $(^nC_0 + {}^nC_1$ + ----------------- $^nC_n)$ - $^nC_0$

= $2^n - 1$ [$^nC_0=1$]

Example: John has 8 friends. In how many ways can he invite one or more of them to dinner?

Ans. John can select one or more than one of his 8 friends.

=> Required number of ways = $2^8 - 1$= 255.

(iv) Number of ways of selecting zero or more things from ‘n’ identical things is given by :- n+1

Example: In how many ways, can zero or more letters be selected form the letters AAAAA?

Ans. Number of ways of :

Selecting zero 'A's = 1

Selecting one 'A's = 1

Selecting two 'A's = 1

Selecting three 'A's = 1

Selecting four 'A's = 1

Selecting five 'A's = 1

=> Required number of ways = 6 [5+1]

Q14 What are generating function ?

Ans Let a_0 , a_1, a_2, …., a_r be a sequence of real numbers. Then the function defined as:

$G(a,z) = a_0 + a_1 z + a_2 z^2 + \ldots + a_r z^r + \ldots = \sum_{n=0}^{\infty} a_n z^n$

Is called Generating function for the given sequence, where z is a variable.

For example, the generating function of the numeric function $a_r = 3^r$ (r >= 0) is

$G(x) = 3^0 + 3^1 x + 3^2 x^2 + \ldots + 3^r x^r + \ldots$ which can be written in closed form as

$$G(x) = \frac{1}{1-3x}$$

Q 15 What is the Pigeonhole Principle ?

Ans If n pigeonholes are occupied by n+1 or more pigeons, then atleast one pigeonhole is occupied by more than one pigeon.

Generalized pigeonhole principle is:- If n pigeonholes are occupied by kn+1 or more pigeons, where k is a positive integer, then at least one pigeonhole is occupied by k+1 or more pigeons.

Q16 What is the principle of inclusion-exclusion?

Ans Let A , B and C be any finite sets. Then according to principle of inclusion and exclusion:-

(1) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(2) $n(A \cup B \cup C) = n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$

Multiple Choice Question

Q1. Which of the following statement is the negation of the statement,
"2 is even and –3 is negative"?
(A) 2 is even and –3 is not negative.
(B) 2 is odd and –3 is not negative.
(C) 2 is even or –3 is not negative.
(D) 2 is odd or –3 is not negative.
Ans:D

Q2 A partial ordered relation is transitive, reflexive and
(A) antisymmetric. **(B)** bisymmetric.
(C) antireflexive. **(D)** asymmetric.
Ans:A

Q3 Let N = {1, 2, 3, ….} be ordered by divisibility, which of the following subset is totally ordered,
(A) 2, 6, 24. **(B)** 3, 5, 15.
(C) 2, 9, 16. **(D)** 4, 15, 30.
Ans:A

Q.4 If B is a Boolean Algebra, then which of the following is true
(A) B is a finite but not complemented lattice.
(B) B is a finite, complemented and distributive lattice.
(C) B is a finite, distributive but not complemented lattice.
(D) B is not distributive lattice.
Ans:B

Q.5 The number of distinguishable permutations of the letters in the word BANANA are,
(A) 60. **(B)** 36.
(C) 20. **(D)** 10.
Ans:A

Q.6 The minimized expression of ABC ABC ABC ABC is
(A) A C. **(B)** BC .
(C) C . **(D)** C.
Ans:C

Q.7 Which of the following pair is not congruent modulo 7?
(A) 10, 24 **(B)** 25, 56
(C) -31, 11 **(D)** -64, -15
Ans:B

Q.8 For a relation R on set A, let M □ m □ , m 1 R ij ij □ □ if aiRaj and 0 otherwise, be the matrix of relation R. If 2 MR =MR then R is,
(A) Symmetric **(B)** Transitive
(C) Antisymmetric **(D)** Reflexive
Ans:B
`

Q.9 If x and y are real numbers then max (x, y) + min (x, y) is equal to
(A) 2x **(B)** 2y
(C) (x+y)/2 **(D)** x+y
Ans:D

Q.10 The sum of the entries in the fourth row of Pascal's triangle is
(A) 8 **(B)** 4
(C) 10 **(D)** 16
Ans:A

Q.11 Which of the following statement is the negation of the statement "2 is even or –3 isnegative"?
(A) 2 is even & -3 is negative **(B)** 2 is odd & -3 is not negative
(C) 2 is odd or –3 is not negative **(D)** 2 is even or –3 is not negative
Ans:B

Q.12 In how many ways can a president and vice president be chosen from a set of 30 candidates?
(A) 820 **(B)** 850
(C) 880 **(D)** 870

Q.13 The relation { (1,2), (1,3), (3,1), (1,1), (3,3), (3,2), (1,4), (4,2), (3,4)} is
(A) Reflexive. **(B)** Transitive.
(C) Symmetric. **(D)** Asymmetric.
Ans:B

Q.14 The expression ą a c is equivalent to

(A) a **(B)** a+c
(C) c **(D)** 1
Ans:B

Q.15 A partial order relation is reflexive, antisymmetric and
(A) Transitive. **(B)** Symmetric.
(C) Bisymmetric. **(D)** Asymmetric.
Ans:A

Q.16 If n is an integer and n2 is odd ,then n is:
(A) even. **(B)** odd.
(C) even or odd. **(D)** prime.
Ans:B

Q.17 In how many ways can 5 balls be chosen so that 2 are red and 3 are black
(A) 910. **(B)** 990.
(C) 980. **(D)** 970.
Ans:B

Q.18 A tree with n vertices has _____ edges
(A) n **(B)** n+1
(C) n-2 **(D)** n-1
Ans:D

Q.19 Which of the following statement is true:
(A) Every graph is not its own sub graph.
(B) The terminal vertex of a graph are of degree two.
(C) A tree with n vertices has n edges.
(D) A single vertex in graph G is a sub graph of G.
Ans:D

Q.20 Pigeonhole principle states that A□ B and A □ B then:
(A) f is not onto **(B)** f is not one-one
(C) f is neither one-one nor onto **(D)** f may be one-one
Ans:B

Q.21 The number of distinct relations on a set of 3 elements is:
(A) 8 **(B)** 9
(C) 18 **(D)** 512

Ans:D

Q.22 A self complemented, distributive lattice is called
(A) Boolean Algebra **(B)** Modular lattice
(C) Complete lattice **(D)** Self dual lattice
Ans:A

Q.23 How many 5-cards consists only of hearts?
(A) 1127 **(B)** 1287
(C) 1487 **(D)** 1687
Ans:B

Q.24 The number of diagonals that can be drawn by joining the vertices of an octagon is:
11
(A) 28 **(B)** 48
(C) 20 **(D)** 24
Ans:C

Q.25 A graph in which all nodes are of equal degrees is known as:
(A) Multigraph **(B)** Regular graph
(C) Complete lattice **(D)** non regular graph
Ans:B

Q.26 Transitivity and irreflexive imply:
(A) Symmetric **(B)** Reflexive
(C) Irreflexive **(D)** Asymmetric
Ans:D

Q.27A binary Tree T has n leaf nodes. The number of nodes of degree 2 in T is:
(A) log n **(B)** n
(C) n-1 **(D)** n+1
Ans:A

Q.28 Push down machine represents:
(A) Type 0 Grammar **(B)** Type 1 grammar
(C) Type-2 grammar **(D)** Type-3 grammar
Ans:C

Q.29 Let * be a Boolean operation defined by

A * B = AB + A B , then A*A is:

(A) A **(B)** B
(C) 0 **(D)** 1
Ans:D

Q.30 In how many ways can a party of 7 persons arrange themselves around a circular table?
(A) 6! **(B)** 7!
(C) 5! **(D)** 7
Ans:A

Q.31 In how many ways can a hungry student choose 3 toppings for his prize from a list of 10delicious possibilities?
(A) 100 **(B)** 120
(C) 110 **(D)** 150
Ans:B

Q.32 A debating team consists of 3 boys and 2 girls. Find the number of ways they can sit in a row?
(A) 120 **(B)** 24
(C) 720 **(D)** 12
Ans:A

Q.33 Suppose v is an isolated vertex in a graph, then the degree of v is:
(A) 0 **(B)** 1
(C) 2 **(D)** 3
Ans:A

Q.34 In an undirected graph the number of nodes with odd degree must be
(A) Zero **(B)** Odd
(C) Prime **(D)** Even
Ans:D

Q.35 Find the number of relations from A = {cat, dog, rat} to B = {male , female}
(A) 64 **(B)** 6
(C) 32 **(D)** 15
Ans:A

Q.36 The number of functions from an m element set to an n element set is:
(A) mn **(B)** m + n
(C) nm **(D)** m * n
Ans:A

Q.37 Which of the following statement is true:
(A) Every graph is not its own subgraph
(B) The terminal vertex of a graph are of degree two.
(C) A tree with n vertices has n edges.
(D) A single vertex in graph G is a subgraph of G.
Ans:D

Q.38 What is the converse of the following assertion?
I stay only if you go.
(A) I stay if you go. **(B)** If you do not go then I do not stay
(C) If I stay then you go. **(D)** If you do not stay then you go.
Ans:B

Q.39 The length of Hamiltonian Path in a connected graph of n vertices is
(A) n–1 **(B)** n
(C) n+1 **(D)** n/2
Ans:A

Q.40A graph with one vertex and no edges is:
(A) multigraph **(B)** digraph
(C) isolated graph **(D)** trivial graph
Ans:D

Q 41 If R is a relation "Less Than" from A = {1,2,3,4} to B = {1,3,5} then RoR-1 is
(A) {(3,3), (3,4), (3,5)}
(B) {(3,1), (5,1), (3,2), (5,2), (5,3), (5,4)}
(C) {(3,3), (3,5), (5,3), (5,5)}
(D) {(1,3), (1,5), (2,3), (2,5), (3,5), (4,5)}
Ans:C

Q.42 How many different words can be formed out of the letters of the word VARANASI?
(A) 64 **(B)** 120
(C) 40320 **(D)** 720

Ans:D

Q.43 Which of the following statement is the negation of the statement "4 is even or -5 is negative"?
(A) 4 is odd and -5 is not negative **(B)** 4 is even or -5 is not negative
(C) 4 is odd or -5 is not negative **(D)** 4 is even and -5 is not negative
Ans:A

Q.44 A complete graph of n vertices should have __________ edges.
(A) n-1 **(B)** n
(C) n(n-1)/2 **(D)** n(n+1)/2
Ans:C

Q.45 A relation that is reflexive, anti-symmetric and transitive is a
(A) function **(B)** equivalence relation
(C) partial order **(D)** None of these
Ans:C

Q.46 A Euler graph is one in which
(A) Only two vertices are of odd degree and rests are even
(B) Only two vertices are of even degree and rests are odd
(C) All the vertices are of odd degree
(D) All the vertices are of even degree
Ans:D

Q.47 What kind of strings is rejected by the following automaton?
(A) All strings with two consecutive zeros
(B) All strings with two consecutive ones
(C) All strings with alternate 1 and 0
(D) None
Ans:B

Q.48 A spanning tree of a graph is one that includes
(A) All the vertices of the graph
(B) All the edges of the graph
(C) Only the vertices of odd degree
(D) Only the vertices of even degree
Ans:A

Q.49 The Boolean expression $A + AB + AB$ is independent to

(A) A **(B)** B

(C) Both A and B **(D)** None

Ans:B

Q.50 Seven (distinct) car accidents occurred in a week. What is the probability that they alloccurred on the same day?

(A) 1 76 **(B)** 7 1 2

(C) 5 1 7 **(D)** 7 1 7

Ans:A

KEY TERMS

Proposition:-A Proposition or a statement or logical sentence is a declarative sentence which is either true or false.

Predicate:- All that is told about the subject in a sentence is called predicate.

compound statement:- Statements or propositional variables can be combined by means of logical connectives (operators) to form a single statement called compound statements.

Biconditional statement or Equivalence:- If p and q are two statements then "p if and only if q" is a compound statement, denoted as $p \leftrightarrow q$ and reffered as a biconditional statement or an equivalence.

Set:- A well defined collection of objects or elements is called a set.

Finite set : A set is called a finite set if the process of counting the elements of that set surely comes to an end. Example is A={2,4,6,8} is a finite set because the number of elements in set A is 4.

Infinite set: A set is called an infinite set if the process of counting the number of elements in that set never ends, ie there are infinite elements in the set. Example is N= set of natural numbers .

Universal set:- In any application of the theory of sets, the members of all sets under investigation usually belong to some fixed large set called the universal set. The universal set is denoted by U.

Subsets:- If every element in a set A is also an element of a set B, then A is called a subset of B. It can be denoted as $A \subset B$. Here B is called Superset of A.

Example: If A={1,2} and B={4,2,1} the A is the subset of B or $A \subset B$.

Null set:- A set having no elements is called a Null set or void set. It is denoted by $\emptyset$.

Union : Let A and B be two sets then union of A and B which is denoted as $A \cup B$ is a set of elements which belongs either to A or to B or to both A and B.

Intersection : Intersection of A and B which is denoted as $A \cap B$ is a set which contains those elements that belong to both A and B.

Difference : Let A and B be two sets. The difference of A and B which is written as A - B, is a set of all those elements of A which do not belongs to B.

Rusell's Paradox:- For a collection to be a set it is necessary that we should be able to decide whether it belongs to the set or not. The assumption that every collection is a set leads to a paradox known as **Russel Paradox.**

Walk : An alternating sequence of vertices and edges is called a ***Walk.*** It is denoted by ***'W'.***

Open Walk : If the end vertices of a walk are different then such a walk is called ***Open Walk.***
Closed Walk : If a walk starts and end with same vertex then such a walk is called closed walk.
Trail : An open walk in a graph G in which no edge is repeated is called a ***Trail.***
Path : An open walk in which no vertex is repeated except the initial and terminal vertex is called a ***Path.***
Circuit : A closed trail is called a ***Circuit.***
Cycle : A closed path is called a ***Cycle.***
Regular Graph : A simple graph G = (V, E) is called a ***Regular Graph*** if degree of each of its vertices are equal.

Complete Graph : A simple graph G = (V, E) is called a ***Complete Graph*** if there is exactly one edge between every pair of distinct vertices. A complete graph with n-vertices is denoted by K_n.
Group:- Associativity, Identity, Inverse.
Subgroup.:- If a nonvoid subset H of a group G is itself a group under the operation of G, we say H is a subgroup of G.
<u>Cyclic Subgroup</u>:- A Subgroup K of a group G is said to be **cyclic subgroup** if there exists an element x ∈ G such that every element of K can be written in the form x^n for some n ∈ Z.
The element x is called generator of K and we write K=<x>
<u>Cyclic Group</u>:- In the case when G=<x>, we say G is cyclic and x is a generator of G. That is, a group G is said to be cyclic if there is an element x ∈ G such that every element of G can be written in the form x^n for the some n ∈ Z.
Alphabets:- The finite non-void set A of symbols is called alphabet or vocabulary.
word or sentence :- If we select some symbols out of the alphabet A to form a finite sequence or a finite string, then what we obtain is called a word or sentence.
Empty word or empty string:- Any string is called empty string, if it is formed by none of the symbols/letters of alphabet A. Empty string is denoted by λ .
Language. Explain ::-The collection of words on an alphabet A is called a language.
Partially ordered set or Poset :- Let R is a relation on a set S satisfying the following three properties:
Reflexive,Antisymmetric ,Transitive , Then R is called a partial order.
Permutation : Permutation means *arrangement* of things. The word *arrangement* is used, if the order of things *is considered.*

Combination: Combination means *selection* of things. The word *selection* is used, when the order of things has *no importance*.

Bibliography

1. Bush, James R. (2003), *Discrete Mathematics Workbook* (Upper Saddle River, NJ: Pearson/Prentice Hall).
2. Ensley, Douglas E.; & Crawley, J. Winston (2006), *Discrete Mathematics: Mathematical Reasoning and Proof with Puzzles, Patterns, and Games* (Hoboken, NJ: John Wiley & Sons).
3. Epp, Susanna S. (2004), *Discrete Mathematics with Applications, Third Edition* (Belmont, CA: Brooks/Cole—Thomson Learning).
4. Feil, Todd; & Krone, Joan (2003), *Essential Discrete Mathematics for Computer Science* (Upper Saddle River, NJ: Pearson/Prentice Hall).
5. Gerstin, Judith L. (2007), *Mathematical Structures for Computer Science: A Modern Approach to Discrete Mathematics, Sixth Edition* (New York: W.H. Freeman).
6. Goodaire, Edgar G.; & Parmenter, Michael M. (2006), *Discrete Mathematics with Graph Theory, Third Edition* (Upper Saddle River, NJ: Pearson/Prentice Hall).
7. Grassmann, Winfried Karl; & Tremblay, Jean-Paul (1996), *Logic and Discrete Mathematics: A Computer Science Perspective* (Upper Saddle River, NJ: Prentice Hall).

Websites:

1. *en.wikipedia.org/wiki/**Discrete_mathematics***
2. *http://home.iitk.ac.in/~arlal/book/mth202.pdf*
3. http://www.cs.columbia.edu/~zeph/3203s04/lectures.html
4. http://www.cs.columbia.edu/~zeph/3203s04/lectures.html