B.Sc. (Part-I) PCM

Model Paper - A

Paper -II (Calculus)

Time allowed : 3 Hrs Max. Marks : 50

Note : Attempt any five question in all. Select at least one from each unit. Each question carry equal marks

UNIT-I

1. (a) Test the convergence of the following series :-

$\frac{x}{1} + \frac{1}{2}\frac{x^2}{3} + \frac{1.3}{2.4}\frac{x^3}{5} + \frac{1.3.5}{2.4.6}\frac{x^4}{7} + \cdots$ ....

(b) Examine the convergence of the following series :-

(i.) $\frac{1}{2} + \frac{\sqrt{2}}{5} + \frac{\sqrt{3}}{8} + \cdots + \frac{\sqrt{n}}{3n-1} + \cdots$

(ii.) $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \frac{4^4x^4}{4!}$

or

2. (a) Test the convergence and absolute convergence of the following series.

$1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \cdots + (-1)^{n-1}\frac{1}{n^p} + \cdots$ ..

(b) Expand $\sin x$ in Maclaurin's series.

UNIT-II

3. (a) If $\frac{2a}{r} = 1 + \cos\theta$, then with usual notations show that $\frac{ds}{d\Psi} = \frac{2a}{sin^3\psi}$

(b) Show that in the parabola $y^2 = 4ax$ the radius of curvature at any point P is $\frac{2\,(SP)^{3/2}}{\sqrt{a}}$ where S is the focus of the parabola.

or

4. (a) IF $x^x, y^y z^z = C$ then prove that $x = y = z$

$\frac{cz}{\partial x dy} = \frac{-1}{x log\, e^x}$

(b) If $u = tan^{-1}\frac{x^3+y^3}{x+y}$, then show that

$x^2\,\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\,\frac{\partial^2 u}{\partial y^2} = \sin 2u\ (1 - 4\sin^2 u)$

UNIT-III

5. (a) Prove that the envelope of the family of parabolas $\sqrt{x/a} + \sqrt{y/b} = 1$ is an astrold when $ab = c^2$ C being constant

(b) Find the maximum and minimum value of function $u = \sin x, \sin y, \sin(x + y)$

or

6. (a) Find the asymptotes of following curve
$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0$

(b) Trace the following curve
$y^2(a^2 + x^2) = x^2(a^2 - x^2)$

UNIT-IV

7. (a) Find the whole length of the cycloid $x = a\, cos^3 t, y = b\, sin^3 t$. Hence find the whole length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$

(b) Find the area of a loop of the curve
$r = a \sin 3\theta$

or

8. (a) Find the area common to the following curves $y^2 = ax$ and $x^2 + y^2 = 4ax$

(b) Find the volume of the spindle shaped solid generated by resolving the following astroid about $x - axy$ $x = a\, cos^3 t,$ $y = a\, sin^3 t$

UNIT-V

9. (a) Change the order of integration of the following integral
$\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} V dx\, dy$

(b) Integrate $r \sin\theta$ over the area of the cardioids $r = a\,(1 + cos\theta)$ about the initial line.

or

10. (a) Evaluate the following integral by changing the order of integration
$\int_0^\infty \int_0^\infty \frac{e^{-y}}{y}\, dxdy$

(b) Evaluate $\int \int \int xyz\, dx\, dy\, dz$ where the region of integration is the complete ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \le 1$

B.Sc. (Part-I) PCM

Model Paper - B

Paper -II (Calculus)

Time allowed : 3 Hrs Max. Marks : 50

Note : Attempt any five question in all. Select atleast one from each unit. Each question carry equal marks

UNIT-I

1. (a) Test the convergence of the following series :-

$\frac{1}{1+x}+\frac{x}{1+x^2}+\frac{x^2}{1+x^3}+\cdots$ ....

(b) Prove that the following of hyper harmonic series is

(i) Convergent if $p > 1$

(ii) Divergent if $p \leq 1$

$\Sigma \frac{1}{n^p} = \frac{1}{2^p} + \ldots \ldots \frac{1}{n^p} + \ldots$

2. (a) Discuss the convergence and absolute convergence of the following series

$\frac{1}{a}-\frac{1}{a+x}+\frac{1}{a+2x}-\frac{1}{a+3x}+--------x > 0$

(b) Expand $log \ (1+x)$ in Maclaurim series.

UNIT-II

3. (a) Find the pedal equation of an ellipse $\frac{l}{r} = 1 + e\ cos\theta; (e < 1)$

(b) Prove that the radius of curvature at any point (x, y) on the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is there time the length of perpendicular from the origin on the tangent at that point.

4. (a) If $V = F\ (x-y, y-z, z-x)$ then prove that

$$\frac{\partial V}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial V}{\partial z} = 0$$

(b) Transform $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in polar coordinates

UNIT-III

5. (a) Show that the envelope of the straigh line joining the extremities of a pair of semi conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$

(b) In a triangle, find a point from which the sum of square of a distance of the vertices is minimum.

Or

6. (a) Show that eight points of intersection of following curve and its asympolotes lie on a rectangular hyper data.
$x^4 - 5x^2y^2 + 4y^4 + x^2 - y^2 + x + y + 1 = 0$

(b) Trace the cissoids $y^2(2a - x) = x^3$

UNIT-IV

7. (a) Find the perimeter of cardioids $r = a(1 + \cos\theta)$. Also show that upper half arc of the cardioids $r = a\,(1 + \cos\theta)$ is bisected by the line $\theta = {}^{\pi}/_{3}$

(b) Find the common area to the circles $r = a\sqrt{2}$ and $r = 2a\ \cos\theta$.

Or

8. (a) Prove that the length of the arc from the vertex to any point on the cycloid $x = a\,(\theta + sin\theta)$, $y = a(1 - \cos\theta)$ is $\sqrt{8ay}$. Also prove that the whole length of an arc of curve 8a.

(b) Find the volume of the solid generated by resolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about
$x - axy$

UNIT-V

9. (a) Evaluate the following integral by changing to polar coordinates $\int_0^{\prime} \int_x^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}dx\,dy$

(b) Evaluate $\iint (x+y)^2\,dx\,dy$ where R is the region of integration given below $\frac{x^2}{a^2}+\frac{y^2}{a^2}=1$

10. (a) If the region A of the integration is the triangle given by $y=0, y=x, x=1$ then show that $\iint_A \sqrt{4x^2-y^2}dx-dy=\frac{1}{3}\left[{}^{\pi}/_3+{}^{\sqrt{3}}/_2\right]$

(b) Find the value $\int\int\int_v x^2dx\,dy\,dz$ where area V is bounded from the following surface $x=0, z=0$ and $x+y+z=a,\ a\succ 0$