<u>Biyani Girls college ,Jaipur</u>

Model Paper-B (B.Sc. I)

Subject:Mathematics

Paper : Discrete Maths

Max Marks: 50 **Max Time: 3 hrs**

Attempt any five questions in all selecting atleast one question from each unit.

Unit I

(1) (i) Prove that the set $(A \cup B) \cap (A' \cap B)'$ is simply the same as the set A itself.

(ii) Prove by principle of mathematical induction that:-

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

(iii) If **If A_1, A_2 and A_3 be finite sets then prove that**

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + |A_1 \cap A_2 \cap A_3|$$

1. (a) If R and S are any relations from the sets A to B and from B to C respectively, then prove that

   (b) Prove that the dual of a lattice is also a lattice.

Unit II

2. (a) Prove that the identity element in a group (G, O) is unique.

   (b) Prove that every field is an integral domain but its converse is not always true

3. (a) Prove that in Boolean algebra B, for all elements a, b, c $\in$ B:-

   ab+bc+ca= (a+b) (b+c) (c+a).

   (b) Express the following Boolean function in its conjunctive normal form:-

   $f(x_1,x_2,x_3) = (x_1+x_2+x_3)(x_1x_2+x_1x_3)$

Unit III

4. (a) Write each of the following in terms of p, q, r and logical connectives

   I will go to a movie or I will not study discrete structures if and only if I am in a good mood.

   (b) Show that $(P \to Q) \vee (Q \to P)$ is a tautology.

   Or

5. (a) Find the generating function for the sequence 0, 2,6, 12, 20, 30, 42,…….?

(b) Solve the following recurrence relation

$a_{n+2}=a_{n+1}+a_n$ n≥ 0 $a_0=0$, $a_1=1$

Unit IV

6. (a) Define the following:-

(i) Mixed graph

(ii) Cycle graph

(iii) Complementary graph

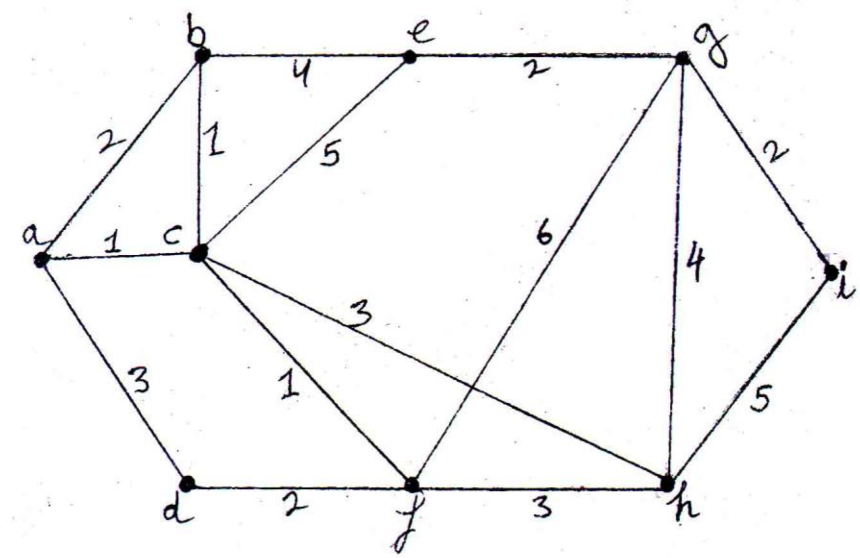
(iv) product of two graphs

(v) Sub graph

(b) Prove that in a complete graph on n vertices (n is odd and ≥3), there are exactly $\frac{n-1}{2}$ edge-disjoint Hamiltonian cycles.

7.

(a) Determine the shortest path and its value between the vertices a and I in the following weighted graph:-



(b) If G is a connected planar graph n vertices, e edges and r regions, then prove that n-e+r=2.

Unit V

8. Prove that a tree on n vertices has exactly n-1 edges.

(a) Define the following:-

(i) Eccentricity of a vertex.

(ii) Centre of a tree.

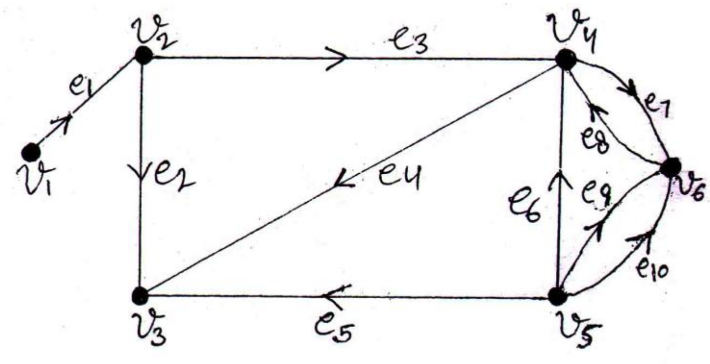
(iii) Binary tree.

(iv) Height of a binary tree.

(v) Spanning tree.

9. (a) Prove that every tree has either one or two centres.

(b) Define the adjacency matrix of a directed graph, and find the adjacency matrix of the following directed graph:-



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Unit I

1. (a) Show that $3^{2n+1} + 2^{n-1}$ is a multiple of 7

(b) If A and B are any two finite sets, then prove that:-

$[A \cup B]$ = [A] + $[B]$ - $[A \cap B]$

(a) Prove that relation R on a non-empty set A is symmetric if and only if R = R^{-1}

(b) Define lattice and prove that the dual of a lattice is also a lattice.

Unit II

2. (a) Define Group. Prove that the set of all integers is an infinite abelian group with respect to ordinary multiplication composition..

(b) Define the following:

(i) Ring

(ii) Ring with zero divisors

(iii) Integral domain

(iv) Field

3. (a) State and prove that De-Morgan's laws in Boolean algebra

(b) Find the disjunctive normal form of the following Boolean function:-

F(x1, x2, x3) = [x1+(x1'+x2')][x1+(x2'.x3')']

Unit III

4(a) U se a truth table to test the validity of the following argument.

If you invest in the Gomermatic Corporation, then you get rich.

You didn't invest in the Gomermatic Corporation.

Therefore, you didn't get rich.

(b) Define Predicate and Quantifier

4. (a) Find the generating function of the following sequences

(i) 0. 1, 2, 3, 4, 5,…….

(ii) 0^2, 1^2, 2^2, $3^{2,\ldots\ldots}$

(b) Solve the following recurrence relation by the method of generating functions:-

(i) $a^r = 3a_{r-1} - 2a_{r-2}$; a_1=5, $a_{2=3}$

Unit IV

5. (a) Prove that the number of vertices of odd degrees in a graph G is always even.

(b) Define the following:

(i) Graph

(ii) Pendent vetex

(ii) Regular Graph

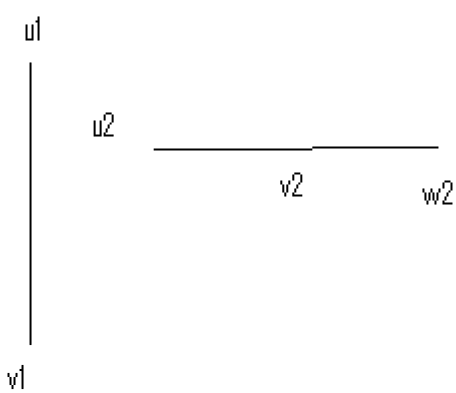
(iii) Complete graph

(iv) Bipartite Graph

6.

(a) Define Planar Graph,Show that the complete bipartite graph $K_{3,3}$is not a olanar Graph.

(b) Define composition of two graphs. For thr following graphs G1 & G2 find their composite graphb G1[G2]



Unit V

7. (a) Define Tree. Prove that a tree on n vertices has exactly n-1 edges.

(b) Define the following:-

(vi) Binary tree.
(vii) Diagraph
(viii) Out-degree and in-degree of a vertex in a diagraph
(ix) Adjacency matrix of graph.
(x) Adjacency matrix of diagraph

8. (a) Prove that a graph G is connected if and only if if it has a spanning tree.

   (b) Prove that every non-trival tree has at least two pendent vertices.