

Biyani Girls college ,Jaipur

Model Paper-A (B.Sc. II)

Subject: Mathematics

Paper : Numerical Analysis

Max Marks: 32

Max Time: 2:30 hrs

Attempt any five questions in all selecting atleast one question from each unit.

Unit 1

Q.1 Prove that

$$(i) \mu\delta = \frac{1}{2} (\Delta + \nabla)$$

$$(ii) 1 + \left(\frac{\delta^2}{2}\right) = \sqrt{1 + \delta^2 \mu^2}$$

Given, $\log 100 = 2$, $\log 101 = 2.0043$, $\log 103 = 2.0128$, $\log 104 = 2.0170$. Find $\log 102$.

Or

(i) Derive the Newton Forward Interpolation Formula.

(ii)

By means of Lagrange's formula, prove that

$$y_1 = y_3 - 0.3 (y_5 - y_{-3}) + 0.2 (y_{-3} - y_{-5}).$$

Unit 2

Q.2 (i)

The values of e^{-x} at $x = 1.72$ to $x = 1.76$ are given in the following table:

| | | | | | |
|-----------|---------|---------|---------|---------|---------|
| $x:$ | 1.72 | 1.73 | 1.74 | 1.75 | 1.76 |
| $e^{-x}:$ | 0.17907 | 0.17728 | 0.17552 | 0.17377 | 0.17204 |

Find the value of $e^{-1.7425}$ using Gauss' forward difference formula.

(ii)

Find the value of y_{15} , using Bessel's formula, if

$$y_{10} = 2854, \quad y_{14} = 3162, \quad y_{18} = 3544, \quad y_{22} = 3992.$$

or

- (i) Find the First Derivative and second Derivative of Newton Backward Interpolation.

Evaluate

$$\int_0^6 \frac{dx}{1+x^2} \text{ by using}$$

- (i) *Simpson's one-third rule*
(ii) *Simpson's three-eighth rule*
(iii) *Trapezoidal rule*

Unit 3

Q.3 (i)

Approximate y and z by using Picard's method for the particular solution of $\frac{dy}{dx} = x + z$, $\frac{dz}{dx} = x - y^2$ given that $y = 2$, $z = 1$ when $x = 0$.

- (i) By Runge Kutta method

Given $\frac{dy}{dx} = y - x$, $y(0) = 2$. Find $y(0.1)$ and $y(0.2)$ correct to f decimal places

Or

Given that $\frac{dy}{dx} = \log_{10}(x + y)$ with the initial condition that $y = 1$ when $x = 0$. Find y for $x = 0.2$ and $x = 0.5$ using Euler's modified formula.

Unit 4

Q.4. A necessary and sufficient condition that a vector $\overrightarrow{F}(t)$ to be constant direction only is $\vec{F} \times \frac{d\vec{F}}{dt}$

- (b) Find the directional derivative of $\phi = 4xz^3 - 3x^2y^2z$ at the point (2, -1, 2) in the direction $2i - sj + 6k$.

or

Verify Stokes Theorem for the function $F = zi + xj + yk$ where C is the unit circle in the x-y plane bounding the hemisphere $z = \sqrt{1 - x^2 - y^2}$

Verify divergence theorem for $F = xyi + z^2j + 2yzk$ on the tetrahedron $x = y = z = 0$, $x + y + z = 1$

Biyani Girls college ,Jaipur

Model Paper-B (B.Sc. II)

Subject: Mathematics

Paper : Numerical Analysis

Max Marks: 32

Max Time: 2:30 hrs

Unit 1

Q.1

Express $y = 2x^3 - 3x^2 + 3x - 10$ in factorial notation and hence show that $\Delta^3 y = 12$.

b)

Estimate the production for 1964 and 1966 from the following data:

or

Prove that the Lagrange's formula can be put in the form

$$P_n(x) = \sum_{r=0}^n \frac{\phi(x) f(x_r)}{(x - x_r) \phi'(x_r)}$$

where

$$\phi(x) = \prod_{r=0}^n (x - x_r)$$

Unit 2

Q.2 (i) *From the following table, find the value of $e^{0.24}$*

Where $x = 0.1, 0.2, 0.3, 0.4, 0.5$

(ii)

Using Bessel's formula, find $f'(7.5)$ from the following table:

| | | | | | | | |
|---------|-------|-------|-------|-------|-------|-------|-------|
| $x:$ | 7.47 | 7.48 | 7.49 | 7.5 | 7.51 | 7.52 | 7.53 |
| $f(x):$ | 0.193 | 0.195 | 0.198 | 0.201 | 0.203 | 0.206 | 0.208 |

or

(a) Use Gauss's forward interpolation formula to find $f(32)$ from the given table

| | | | | |
|-------|--------|--------|--------|--------|
| X: | 25 | 30 | 35 | 40 |
| F(x): | 0.2707 | 0.3027 | 0.3386 | 0.3794 |

(b)

If third differences are constant, prove that

$$y_{x+\frac{1}{2}} = \frac{1}{2}(y_x + y_{x+1}) - \frac{1}{16}(\Delta^2 y_{x-1} + \Delta^2 y_x)$$

Unit 3

Q.3 Solve the following system of equations :

$$10x_1 + 2x_2 + x_3 = 9$$

$$2x_1 + 20x_2 - 2x_3 = -44$$

$$-2x_1 + 3x_2 + 10x_3 = 22$$

Use Gauss Jordan Method.

(b) Solve the following system of equation by Jacobi Method.

$$83x_1 + 11x_2 - 4x_3 = 95$$

$$7x_1 + 52x_2 + 13x_3 = 104$$

$$3x_1 + 8x_2 + 29x_3 = 71$$

Or

(a) Given the differential eqn. $\frac{dy}{dx} = \frac{x^2}{y^2 + 1}$

with the initial condition $y = 0$ when $x = 0$. Use Picard's method to obtain y for $x = 0.25, 0.5$ and 1.0 correct to three decimal places.

(b) Using Euler's modified method, obtain a solution of the equation $\frac{dy}{dx} = x + |\sqrt{y}|$ with initial conditions $y = 1$ at $x = 0$ for the range $0 \leq x \leq 0.6$ in the step of 0.2. Correct upto four place of decimals.

Unit 4

Q.4(a) If $\vec{r} \times \frac{d\vec{r}}{dt} = 0$ then show that r is a constant vector.

Prove that $\text{curl}(u\vec{a}) = (\text{grad } u) \times \vec{a} + u \text{curl } \vec{a}$

If $\frac{d^2r}{dt^2} = -n^2r$, then find the value of $\left| \frac{d\vec{r}}{dt} \right|$

Or

Verify Gauss divergence theorem for the function $F = 4xzi - y^2j + yzk$ taken over the cube bounded by $x=0, x=1, y=0, y=1, z=0, z=1$.

Use Green's theorem to evaluate :

$$\int \{(y - \sin x)dx + \cos x dy\}$$

Where c is a triangle enclosed by the lines $y=0, x=\pi/2$ and $y=2x/\pi$