B.Sc (II) PCM
Paper-I Set A
Real Analysis and Metric Space

Time: 2:30 **Maximum Marks: 50**

Unit I

1. (a) If p and q re rational and irrational numbers respectively, then (i) p+q and (ii) pq are irrational numbers.

   (b) Prove that n ordered field is an infinite field.

2. (a) The set Q of rational numbers is not a complete ordered field.

   (b) Every infinite bounded set has at least one limit point.

Unit II

3 (a) Define cauchy's sequence. Every Cauchy sequence is bounded.

(b) Prove that the sequence $\{x_n\}$ where $X_n = \frac{2n-7}{3n+2}$, for all n belong to N

(i) is monotonic increasing

(ii) is bounded.

(iii) lim x_n= 2/3

4. (a) If a function is continuous on [a,b] then it is bounded in that interval.

   (b) If a function f is continuous on [a,b] then it attains its supermum and infimum at least once in [a,b].

Unit III

5. (a) Prove that the following simultaneous limit does not exist lim $_{\substack{x\to 0\\ y\to 0}} \frac{xy3}{x^2+y^6}$

   (b) Let f be real valued bounded on [a,b]. then prove that f is R-integrable over [a,b] iff given $\in > 0$ there exists a partition P of [a,b] such that $0 \leq U(f,p) - L(f,p) < \in$

6. (a) If $f(x) = x, x \in [0,1]$ then show that f is R-integrable on [0,1] and that

$$\int_0^1 x dx = 1/2$$

   (b) if $f \in R[a,b]$ and if there exists a primitive function $\emptyset$ on interval [a,b] then

$$\int_a^b f(x)dx = \emptyset(b) - \emptyset(a)$$

Unit IV

7. (a) Test for uniform convergence the sequence $f_n(x) = nx(1-x)^n \; when \; 0 \leq x \leq 1$

(b) difine metric space .show that let X be a metric space with distance function d and let A be a non empty subset of X then for any x,y$\in$ X

$|d(x, A) - d(y, A)| \leq d(x, y)$.

8. (a) Union of two bounded subsets of a metric space is also bounded.

(b) A subset A of a metric space X is closed iff$\bar{A}$ =A.

Unit V

(9) (a) Every convergent sequence in a metric space X is bounded.

(b) Every convergent sequence in a metric space is a Cauchy sequence but the converse is not True.

(10) (a) Every non empty closed subset of a compact metric space is compact.

(b) let (X,d) be a complet a metric and (A,d) be subspace of (X,d) then A is complete.