# B.Sc (II) PCM
# Paper-I set B
# Real Analysis and Metric Space

**Time: 2:30** **Maximum Marks: 50**

## Unit I

1. (a) Prove that $\sqrt{2}$ is not a rational number.

   (b) Between two different real numbers there lie an infinite number of rational.

2. (a) Prove that the intersection of finite collection of open sets is an open set.

   (b) short notes (i) closure (ii) limit point of set (iii) open set

## Unit II

3 (a) prove that every bounded sequence has at least one limit point.

   (b) Prove that if $\{x_n\}$ is a convergent sequence then its limit is unique.

4. (a) If a function is continuous on [a,b] then it is bounded in that interval.

   (b) A function which is continuous at a point a of its domain in accordance with Cauchy's definition is also continuous by Heine's definition and conversely.

## Unit III

5. (a) Prove that if a function is continuous on[a,b] then it is bounded in that interval

   (b) Let f be real valued bounded on [a,b]. then prove that f is R-integrable over [a,b] iff given $\in > 0$ there exists a partition P of [a,b] such that $0 \le U(f,p) - L(f,p) < \in$

6. (a) If $f(x) = x^2, x \in [0,a]$ then show that f is R-integrable on [0,a] and that

$$\int_0^a f(x)dx = a^3/3$$

   (b) let f be a continuous function defined on [a,b]. prove that f is R-integrable over [a,b].

## Unit IV

**7.** (a) Test for uniform convergence the series $\sum_{n=1}^{\infty} xe^{-nx}$ at x=0.

(b) Prove that if $f_n$ be a sequence of continuous functions defined on a set E and it converges uniformly to a function f(x) on E then f(x) is continuous on E

8. (a) Prove that in a metric space every open sphere is an open set..

   (b) in a metric space the union of an arbitrary collection of open sets is open.

## Unit V

(9 ) (a) short notes (i) complete metric Space (ii) Cauchy sequence (iii) metric space

(b) Every convergent sequence in a metric space is a Cauchy sequence but the converse is not True.

(10) (a) A subset A of R is compact iff A is bounded and closed

(b) Every non empty closed subset of a compact metric space is compact..