B.Sc. III Year (PCM)
Model Paper - A
Paper -I (Abstract Algebra)

Time allowed : 3 Hrs Max. Marks : 50

UNIT -I

Q.1 (a) Let a binary operation * defined on a set
$G = \{(a, b)/a, b \in R \text{ and } a \neq 0\}$ such that
$(a, b) * (c, d) = (ac, bc + d), \forall (a, b), (cd) \in G$
Then show that $(G,*)$ is a group.

(b) Let $\rho = \begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8\,9 \\ 7\,8\,9\,6\,4\,5\,2\,3\,1 \end{pmatrix}$
$\sigma = (1\ 3\ 4)\ (5\ 6)\ (2\ 7\ 8\ 9)$
Then find $\sigma^{-1}\rho\sigma$
Show ρ is multiplication of disjoint cycle. ρ is even or odd permutation ? and also find its order.

Or

Q.2(a) Prove that a non-empty subset H of a group G is subgroup iff $a \in H, b \in H \Rightarrow ab^{-1} \in H$

(b) If H be a subgroup of G then prove that any two right (left) cosets of H are either identical or disjoint.

UNIT-II

Q.3(a) Define isomorphism on a group. State and prove cayley's theorem.

(b) Prove that :-

(i) Homomorphism image of a commutative group is again commutative.

(ii) Homomorphic image of a cyclic group is again cyclic.

Or

Q.4(a) Define quotient group. If $G = (Z, +), H = (4Z, +)$ then find quotient group $\frac{G}{H}$. Find composition table for $\frac{G}{H}$.

(b) Define normal subgroup of a group. If H is a subgroup of a group G and N is a normal subgroup of G then prove that $H \cap N$ is a normal subgroup of H.

UNIT-III

Q.5(a) For a ring R in which $a^2 = a, \forall a \in R$, prove that

(i) $a + a = 0, \forall a \in R$
(ii) $a + b = 0 \Rightarrow a = b, b \in R$
(iii) R is a commutative ring

(b) The necessary and sufficient condition for a non empty subset K of a field F to be a subfield are.

(i.) $a \in K, b \in K \Rightarrow a - b \in K$
(ii.) $a \in K, 0 \neq b \in K \Rightarrow ab^{-1} \in K$

Or

Q.6.(a) Prove that every field is an integral domain but the converse is not necessarily true.

(b) Prove that the field $\langle Q, +, \cdot \rangle$ of rational numbers is prime field.

UNIT-IV

Q.7(a) Prove that the ring $\langle Z, +, \cdot \rangle$ of integers is a principal ideal domain.

(b) If , I_1 and I_2 be two ideals of a ring R, then prove that $I_1 + I_2 = \{a_1 a_2 / a_1 \in I_1, a_2 \in I_2\}$ is an ideal of R containing both I_1 and I_2.

Or

Q.8(a) Prove that the metrix set $V = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} / a, b \in R \right\}$ is a vector space over the field R of real numbers with respect to matrix addition and matrix scalar multiplication.

(b) Define a vector subspace. If V is a vector space over the field F and v_1 and v_2 are fixed elements of V then show that the set

$S = \{\alpha v_1 + \beta v_2 / \alpha, \beta \in F\}$ is a vector subspace.

UNIT-V

9 (a) Define linear spam. Prove that the linear spam L(S) of subset S of a vector space V(F) is the smallest subspace of V(F) containing S.

(b) Prove that the necessary and sufficient conditions for a vector space V(F) to be the direct sum of two of its subspaces U(F) and W(F) are :-

(i) V=U+W (ii) $U \cap W = \{0\}$

Or

10 (a) Prove that every finite dimensional vector space has a basis.

(b) If W be a subspace of a finite dimensional vector space V (F) then prove that

$dim \left({}^{V}/_{W} \right) = \dim V - \dim W$

B.Sc. III Year (PCM)

Model Paper - B

Paper -I (Abstract Algebra)

Time allowed : 3 Hrs Max. Marks : 50

UNIT -I

Q.1(a) Define the order of an element of a group. Prove that n is order of an element a of group G then order of a^p is also n where p and n are relatively prime.

(b) Prove that set of all even permutation of set A_n is group with $\frac{n!}{2}$ order.

Q.2 (a) Define subgroup. State and prove legranges theorem .

or

(b) If H is subgroup of G and (G:H) =2 then prove that $aH = Ha, \forall a \in G$

UNIT -II

Q.3(a) Let f be a homomorphism from group G to G' then f is one-one iff ker f ={e} where e is identify of G.

(b) If H and K are two normal subgroup of G then prove that HK is normal subgroup of G

Or

Q.4 (a) Prove that intersection of any two normal subgroup of a group

(b) Prove that every homomorphism image of a G is isomorphic to some quotient group of G.

UNIT -III

Q.5 (a) Prove that the set

$R = \{m + n\sqrt{2}/m, n \in \mathrm{Z}\}$

Is a ring with respect to ordinary addition and multiplication of real numbers. Is it a field.

(b) Define characteristic of a field and prove that characteristic of a field is either zero or a prime number.

Q.6 (a) Define subring and prove that a nonempty subset S of a ring R is a subring of R iff.

i) $a \in s, b \in S \Rightarrow a - b \in S$

ii) $a \in S, b \in S \Rightarrow ab \in S$

(b) Prove that every ring can be embedded in a ring with unity.

UNIT -IV

UNIT -IV

Q.7(a) Define a principal ideal ring and principal ideal domain prove that the ring $(z, +, .)$ of integers is a principal ideal ring and principal ideal domain.

(b) Prove that an ideal I of a commutative ring R with unity is prime iff $R/_I$ is an integral domain.

Or

Q. 8(a) Let V be a vector space over a field F. Than prove that

i) $a.0 = 0, \forall a \in F$ ii) $0.v = 0, \forall\ a \in V$

iii) $(-a).v = a(-v) = -(a.v);\ \forall\ a \in F, v \in F$

iv) $a.v = 0\ \Rightarrow either\ a = 0$ or $v = 0$

(b) Prove that a non-empty subset W of a vector space V over a field F is a subspace of $V(F)$ iff $av + \beta v \in W$ for all $\alpha, \beta \in F$ and for all $u, v \in W$

UNIT -V

Q.9(a) For which value of k will the vector $u = (5, k, 7) \in V_3(R)$ be a linear combination of vectors $u_1 = (1,5,3)$ and $u_2 = (3,2,1)$

(b) If S and T are subspaces of a vector space $V(F)$ then prove that the set

$S + T = \left\{s + \frac{t}{s} \in s, t \in T\right\}$

or

Q.10 (a) Prove that if $\{u_1 u_2, u_3\}$ is a basis for $V_3(R)$, then $\{u_1 + u_2,\ u_1 + u_3,\ u_2 + u_3\ \}$ is also a basis of $V_3(R)$

(b) If W_1 and W_2 are finite dimensional subspaces of a vector space, then.

dim $(W_1) +$ dim $(W_2) = dim\ \ (W_1 + W_2) + dim\ (W_1 \cap W_2\ )$