B.Sc (III) PCM
Paper-II set A
Complex Analysis

Time: 2:30 **Maximum Marks: 50**

Unit I

1. (a) Show that a sterographic projection projects circles into circles or straight lines.

   (b) Define Analytic function. State and prove the necessary condition for f(z) to be analytic.

2. (a) Derive the polor form of the Cauchy Riemann Equations.

   (b) Show that the function f(z) = u+iv where $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, (z \neq 0), f(0) = 0$

   Is continuous and that the Cauchy Riemanne equation are satisfied at the oigin .yet $f'(0)$ does not exist.

Unit II

3. (a) If f(z) is analytic with a continuous derivative in a simply connected domain G and C is a closed contour lying in G then $\int_c f(z)dz = 0$.

   (c) Verify cauchy's theorem for the function $5\sin 2z$, if C is the square with vertices at $1 \mp i, -1 \mp i$.

4. (a) Prove that The derivative of an analytic function is itself an analytic function.

   (b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C ia a circle $|z| = 3$

Unit III

5. (a) Find the radii of convergence of the following power series:$\sum \frac{-1^n}{n}(z-2i)^n$

   (b) State and prove that the taylor theorem.

6. Define laurent's theorem (a) expand the function $f(z) = \frac{1}{(z+1)(z+3)}$ in the powers of (z+1) valid in the region $0 < |z+1| < 2$.

Unit –IV

7. (a) Define singularities and explain its kind with examples.

   (b) prove that Let a be isolated singularity of f(z) and if $|f(z)|$is bounded in some deleted neighborhood of a , then a is a removable singularity.

8. (a) Find the residues of

$\frac{z^2}{(z-1)(z-2)(z-3)}, at\ z = 1,2,3\ and\ infinity\ and\ show\ that\ their\ sum\ is\ zero.$

(b) suppose that f(z) and g(z) are analytic inside and on a simple closed contour C with $|g(z)| < |f(z)|\ inside\ C$

Unit V

9 (a) Evaluate $\int_0^{\pi} \frac{a d\theta}{a^2+\sin^2\theta}, (a > 0)$

(b)Show that the function $f(z) = \frac{1}{a} + \frac{z}{a^2} + \frac{z^2}{a^3} \ldots \ldots ..$ can be continued analytically outside the circle of convergence.

10. Define conformal mapping . Explain sufficient condition for w=f(z) to represent a conformal mapping