B.Sc (III) PCM
Paper-II Set B
Complex Analysis

Time: 2:30 **Maximum Marks: 50**

Unit I

1. (a) Define Sterographic projection.

   (b) State and prove the sufficient condition for f(z) be analytic.

2. (a) If f(z) = u +iv is an analytic function of z = x+ iy and $u - v = e^x(cosy - siny)$ find f(z) in terms of z.

   (b) Prove that the function $u(x, iy) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1)$is harmonic also determine the harmonic conjugate and find the corresponding f(z) in term of z

Unit 2

3. (a) Let f(z) be a single valued analytic function in a simple connected domain G, if$a, b \in G, then \int_a^b f(z)dz = \emptyset(b) - \emptyset(a)$, where $\emptyset(z)$ is an indefinite integral of f(z).

   (b) Prove that $\int_c \frac{dz}{z-a} = 2\pi i \,, where\ C\ is\ given\ by\ the\ equation\ |z-a| = R.$

4. (a) Prove that if f(z) is analytic function in a simply connected domain G . and z_0 is any point of G $f(z_0) = \frac{1}{2\pi i}\int_c \frac{f(z)}{(z-z_{0)}} dz$

   (b) State and prove Poission integral formula.

Unit-3

5. (a) State and prove laurent's theorem.

   (b) Expand the function $f(z) = \frac{1}{z^2-3z+2}$ valid in the regions:

   (i) $|z| < 1$ $(ii) 1 < |z| < 2$ $(iii)\ |z| > 2$

6. (a) A power series represents an analytic function inside its circle of convergence.

   (b) find the radii of convergence of the following power series: $\sum \frac{n\sqrt{2}+i}{1-2in} z^n$

Unit-4

7. (a) Prove that the necessary and sufficient condition for an isolated singularity z= a to be a pole of function f(z) is that $|f(z)| \to \infty \, as \; z \to a \; is \; any \; manner.$

   (b) find the location and nature of the singularities of the function $f(z) = \frac{1}{z(e^z - 1)}$.

8. (a) state and prove Rouche's theorem .

   (b) Evaluate the integral $\frac{1}{2\pi i} \int_C \frac{e^{zt}}{z^2(z^2+2z+2)} dz \; around \; the \; circle \; C: |z| = 3.$

Unit -5

9. **(a)** Prove by contour integration that $\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx = \pi log2$

   (b) state and prove that Uniqueness of analytic continuation.

10. (a) In the Transformation $z = \frac{i-w}{i+w}$, show that the positive half of the w-plane given by $v \geq 0 \; corresponds \; to \; the \; circle \; |z| \leq 1 \; in \; the \; z - plane.$

   (b) Show that the transformation $w = \frac{2z+3}{z-4} \; maps \; the \; circle \; x^2 + y^2 - 4x = 0 \; into \; the \; straight \; line \; 4u + 3 = 0.$