M.Sc. (F) Mathematics
Model Paper - A
Paper -I (Analysis and Advanced Calculus)

Time allowed : 3 Hrs Max. Marks : 100

Note : Attempt any five question in all. Select at least one from each unit. Each question carry equal marks

UNIT-I

1. (a) Define normed linear space with example. If X be a normed linear space and $x, y \in X$ than show that $\|x\| - \|y\|/\leqslant \ \|x-y\|$
   (b) Define bounded linear transformation. Prove that a linear mapping T of a normed linear space E into a normed linear space F is continuous if T is bounded.

or

2. (a) Define weak and strong convergence. Show that in a finite dimensional space the motions of weal and strong convergences are equivalent.
   (b) Define equivalent norms. Prove that on a finite dimensional linear spaces on X all norms are equivalent.

UNIT-II

3. (a) State and prove closed graph theorem.
   (b) Let B be a banach spare and N be a normed linear space over the same field and let $\{T_i\}$ is a non empty set of continuous linear transformation of B into N with the property that $\{T_i(x)\}$ is a bounded .... of N for each vector in B then $\|T_i\|$ is a bounded set of real numbers.

or

4. (a) Define inner product space and prove that cauchy schwarz inequality for an inner product space.
   (b) Define Hilbert space with example.
   Prove that a closed convex subset M of a Hilbert space H contains a unique vector of smallest norm.

UNIT-III

5. (a) Let M and N are closed linear subspaces of a Hilbert space H s.t. M+N, then prove that the linear subspace M+N is also closed.
   (b) If $\{e, e_2\_\_\_\_\_\_\_\_ e_n\}$ be a finite orthonormal set in a helbert space H then prove that
   $\sum_{i=1}^{n} |< x, ei >|^2 \leq \|x\|^2$ for each vector $x \in H$

or

6. (a) State and prove piesz representation theorem for Hilbert space.

(b) An operator T on a Hilbert space H is unilary iff T is an isometric isomorphism of H onto itself.

UNIT-IV

7. (a) If P is a projection on a Hilbert space H with range M and null space N, then prove that M+N if and only if P is self ad joint and in that case prove that $N = M^{\perp}$

   (b) Define invariance of a closed linear space of a flibert space under an operator T. Prove that a closed linear subspace M of a Hilbert space H is invariant under an operator T iff $M^{\perp}$ is invariant under T.

or

8. (a) If $f: R \to Y$ be a continuous function and f is differentiable on the interval $[a, b]$ of R, $\|(t)\| \leq M$

   for all $t \in (a, b)$ then prove that $\|f\ (b) - f(b)\| \leq M\ (b - a)$

   (b) Let X and Y be banach spaces pver tje same field of scalecs K and U be an open subtract of X. If $F: U \to Y$ is differentiable at x ∈U, then all the directional derivatives of f exist at x and

$$D_a\, f(x) = Df(x)a$$

9. (a) If I be an open internal in R containing [0,1] and $v: I \to$ X is $(n + 1)$ times continuously differentiable function of a single variable t then peave that

$$v\ (1) - v(0) - \ v'(0) - \frac{v''(0)}{2!} - - \frac{v^n(0)}{n!} = \int_0^{\prime} \frac{(1-t)^n}{n!}\ v^{n+1}\ (t)dt$$

10. State and prove existence theorem for approximate solution.

M.Sc. (F) Mathematics

Model Paper - B

Paper -I (Analysis and Advanced Calculus)

Time allowed : 3 Hrs Max. Marks : 100

Note : Attempt any five question in all. Select at least one from each unit. Each question carry equal marks

UNIT-I

1. (a) If M be a closed linear subspace of a normed linear space N and the norm of the corct $x + M$ in the quotient space $\frac{N}{M}$ is defined as

$\|x + M\| = Inf.\{\|x + m\|/m \in M\}$ then $\frac{N}{M}$ is a normed linear space.

(b) Let N and N' be normed linear spaces and $T: N \to N'$ be a linear Transformation. Then T^{-1} exists and is continuous on its domain iff there exist $k > 0\ s.t.$

$$k\|x\| \leq T\ \|x\|, \forall x \in N$$

or

2. Prove that a normed linear space E is a banach space iff every absolutely convergent series in E converges in ∈. Also show that

$$\left\| \sum_{n=1}^{\infty} xn \right\| \leq \sum_{n=1}^{\infty} \|x_n\|$$

UNIT-II

3. (a) State and prove open mapping theorem

(b) If M is a closed linear subspace of a normed linear space N and x_0 is a vector not in M then ∃ a functional f_0 in $N\ s.t.\ f_0\ (M) = 0$ and $f_0(x_0) \neq 0$

or

4. (a) Define inner product space with example. If $\langle x, y\rangle$ is an inner product on a linear space ∈ than prove that $\|x\| = \langle x, x\rangle^{1/2}$ define a norm on ∈

(b) Let M be a closed linear subspace of a Hilbert space H. Let x be a vector not in M and let d be the distance from x to M. Then there exist a unique vector y_o in M s.t $\|x - y_o\|\ d$

UNIT-III

5. (a) State and prove projection theorem in Hilbert space.
   (b) Let H be a Hilbert space and f be an arbitrary functional in H, then there exists a unique vector y in H s.t. $f = f_y\ i.e. f(x) = \langle x, y\rangle, \forall x \in H,$
6. If $\{e_1, e_2 \ldots\ldots e_n\}$ be a finite orthonormal set in a Hilbert space H and x is any vector in H then prove that $\left\| x - \sum_{i=1}^{n} \lambda i e i \right\|$ has its minimum value when $\lambda_i = \langle xi, ei\rangle$ and $\sum_{i=1}^{n} |< x, ei >|^2 \leq \|x\|^2$ also show that $x - \sum_{i=1}^{n} < x, ei > e_i \perp e_j$

UNIT-IV

7. (a) If P is the projection on the closed subspace M of H, then $x \in M$ iff $P_x = x$

   (b) If T is normal, then prove that the eigen spaces corresponding to distinct eigen values of T are pair-wise normal.

or

8. (a) Let f be a continuous function on an open subset U of a Banach space X into a Banach space Y. Let a and b be sub distinct points of Us.t. the segment [a,b] is contained in U, and F be differentiable at all points of [a,b]. Then prove that
   $\|f(b) - f(a)\| \leq \|b - a\|\ sub\ \{\|Df(x)\|/x \in [a, b]\}$

   (b) If T is a normal operator on a Hilbert space H, then x is an eigen vector of T with eigen value λ iff x is an eigenvector of T with eigen value $\bar{\lambda}$

UNIT-V

9. (a) If $f : W \to Y$ is twice differentiable at the point $a \in W$ then prove that the second derivative

   $D^2 f(a) \in L\ (XXX, Y)$ *is pilinear symmetric map.*

   (b) State and prove implicit function theorem.

or

10. Define regulated function. Let f be a regulated function on a compact interval $[a, b]$ of R into a Banach space X over A and g be a continuous linear map of X into a Banach space Y over A. Then got is regulated and $\int_a^b (\text{got}) = g\left(\int_a^b f\right)$