<u>Biyani Girls college ,Jaipur</u>

Model Paper-A (M.Sc. I)

Subject:Mathematics

Paper : VIII (Integral transform and integral equation)

Max Marks: 100 **Max Time: 2:30 hrs**

Attempt any five questions in all selecting atleast one question from each unit.

<u>UNIT-I</u>

1. (a) $Show that: L\left\{\int_0^x \frac{1-e^{-u}}{u} du ; p\right\} = \frac{1}{p}\log\left(1+\frac{1}{p}\right) where\, L\{f(x);p\}$ stands for Laplace transform. 10

(b)Evaluate Lapalce transform of $J_1(x)$,where $J_1(x)$ is the Bessel's function of order one .Hence evaluate Lapalce transform of $xJ_1(ax)$ $(a \neq 0)$. 10

2. (a) State and prove convolution theorem for Laplace Transform and apply it to find $L^{-1}\left\{\frac{p^2}{(p^2+a^2)^2}\right\}$ 2+3+5

(b) State complex inversion formula for inverse Laplace Transform and use it to find $L^{-1}\left\{\frac{p}{(p+1)^3(p-1)^2}\right\}$ 2+8

<u>UNIT-II</u>

3. (a)Find the Fourier Transfrom of f(x) where:

$$f(x) = \begin{cases} 1-x^2, & |x|<1 \\ 0 & |x|>1 \end{cases}$$

and hence evaluate

$$\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right)\cos\left(\frac{x}{2}\right)dx \qquad 4+6$$

(b) State Parseval's identity for Fourier cosine transform. Use this identity to evaluate

$$\int_0^\infty \left(\frac{\sin at}{t(a^2+t^2)}\right) dt \qquad 2+8$$

4. (a) If $n \in N$, then prove that :- $m\left\{\left(x\frac{d}{dx}\right)^n f(x);p\right\} = (-p)^n F(p)$ where $F(p) = m\{f(x);p\}$ and

hence deduce that $m\left\{x^2\frac{d^2 f}{dx^2} + x\frac{df}{dx};p\right\} = (p)^2 F(p)$ $\qquad 8+2$

(b) If $F(p), G(p)$ are Mellin transform of functions f(x) and g(x) then find Mellien transform of

$x^\lambda \int_0^\infty u^\mu f\left(\frac{x}{\mu}\right) g(u)\,du$ (where λ *and* μ are constants) and hence deduce that

$$m^{-1}\{F(p)G(p);x\} = \int_0^\infty f\left(\frac{x}{4}\right) g(u)\frac{du}{u} \qquad 8+2$$

UNIT-III

5a. State and prove Parseval theorem for Hankel transform. 2+8

b. Find the Hankel transform of $t\frac{d^2y}{dt^2} + \frac{dy}{dt} + 4ty = 0$ *having given that* $y(0) = 3$ *and* $y'(0) = 0$. 10

6a. Convert the differential equation $\frac{d^2y}{dx^2} + \lambda y = 0$ with conditions y(0)=0, y(1)=0 into Fredholm integral equation of second kind. Also recover the original differential equation from the integral equation so obtained. 8+2

b. Find the eigen values and eigen functions of the homogenous integral equation:-

$$g(x) = \lambda\int_1^2 \left(xt + \frac{1}{xt}\right) g(t)\,dt \qquad 10$$

UNIT-IV

7a. Form an integral equation correspondingto the differential equation:-

$\frac{d^3y}{dx^3} + x\frac{d^2y}{dx^2} + (x^2 - x)y = xe^x + 1$ with initial condition $y(0)=1=y^1(0)$ and $y''(0)=0$. Also name the integral equation so obtained. 8+2

b. Find the eigrn value and eigen functions for the integral equation

$$g(x) = \lambda \int_{-1}^{1} (5xt^3 + 4x^2 t + 3xt) g(t) dt$$ 10

8a. Solve the integral equation :-

$g(x) = L + \lambda \int_{-1}^{1} (1 - 3xt) g(t) dt$ For what value of λ does the solution not exist? 8+2

b. Solve the integral equation :

$$g(x) = \cos x - x - 2 + \int_{0}^{1} (t - x) g(t) dt$$ 10

UNIT-V

9a. State Hilbert –Schmidt theorem and use it to solve:

$$g(x) = (x+1)^2 + \int_{-1}^{1} (xt + x^2 t^2) g(t) dt$$ 3+17

10a. Using Fredholm First theorem Solve the integral equation

$$g(x) = x + \int_{0}^{1} (4xt - x^2) g(t) dt$$ 10

b. Solve by Laplace transform method, the integral equation

$$g(x) = f(x) + \int_{0}^{x} e^{x-t} g(t) dt$$ 10