Biyani Girls college ,Jaipur

Model Paper-B (M.Sc. I)

Subject:Mathematics

Paper : VIII (Integral transform and integral equation)

Max Marks: 100 **Max Time: 2:30 hrs**

Attempt any five questions in all selecting atleast one question from each unit.

UNIT-I

1. (a) Find the Laplace Transform of $\sin\sqrt{x}$ and hence obtain the Laplace Transform of $\frac{\cos\sqrt{x}}{\sqrt{x}}$.

7+3

(b) Prove that:- $L\{x^{\nu/2}J_\nu(2\sqrt{ax});p\} = a^{\nu/2}p^{-\nu-1}e^{-a/p}$ and hence deduce that :-

$$L\left\{x^{\nu/2}e^{ax}\int_0^\infty t^{-\nu/2}e^{-at}J_\nu(2\sqrt{xt})f(t)\,dt;p\right\} = (p-a)^{-\nu-1}L\{f(t);a+(p-a)^{-1}\}$$ 7 + 3

2a. Using Laplace Transform prove that $\int_0^\infty \cos x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$ 10

b. Using complex inversion formula for inverse Laplace Transform and find

$L^{-1}\left\{\frac{1}{(P+1)(P-2)^2}\right\}$ 10

UNIT-II

3. (a) Find $f(x)$ if its Fourier sine transform is $\frac{p}{1+p^2}$ 10

(b) Find Fourier transform of $e^{-a^2x^2}$, hence obtain Fourier cosine transform of $\cos(x^2/2)$.

8+2

4a. Find Mellin transform of $x^\alpha(1+x^\alpha)^{-b}$. Mention also the condition of validity. 8+2

b. State and prove the convolution theorem for the Fourier transform. 2+8

UNIT-III

5. a Obtain the Hankel transform of $x^{v}H(a-x)$ and $x^{v}H(a-x)$ $(v>-\frac{1}{2}$ and H(x) is the Heaviside unit step function). Hence prove that:-

$$xH_{v}\{x^{-2}J_{v}(ax);p\} = \begin{cases} \frac{1}{2v}\left(\frac{p}{a}\right)^{v} & 0<p<a \\ \frac{1}{2v}\left(\frac{a}{p}\right)^{v} & p>a \end{cases}$$

3+2+5

b. If $F_{v}(p)$ is the Hankel transform of order ‘v’ of f(x), then prove that :-

$H_{v}\{f'(x);p\} = -\frac{p}{2v}\left[(v+1)F_{v-1}(p)-(v-1)F_{v+1}(p)\right]$ where $xf(x)\to 0\, as\, x\to 0\, or\, \infty$ Hence find $H_{1}\{f'(x);p\},\ where\ f(x)=\frac{e^{-ax}}{x}$

7+3

6.a find the bounded solution of the equation $x^{2}\frac{d^{2}y}{dx^{2}}+x\frac{dy}{dx}+(x^{2}-1)y=0$ given that y(1)=2 10

b. solve the diffusion equation $\frac{\partial U}{\partial t}=k\frac{\partial^{2}U}{\partial x^{2}}(-\infty<x<\infty)$ subject to condition that U=f(x) at t=0, f(x) being a given function of x. 10

UNIT-IV

7. Find Fredholm and Volterra integral equations of first and second kinds giving an example for each. Convert the problem $y''+\lambda y=0;\ y(0)=y'(0)\, and\, y(\pi)=y'(\pi)$

To an integral equation also show that the integral equation :

$$g(x)=\lambda\int_{0}^{1}(3x-2)g(t)dt\ \ has\, no\, eigen\, value\, and\, eigen\ function$$

6+8+6

8a.Using resolvent Kernal method solve the integral equation :-

$$g(x)=\frac{3}{2}e^{x}-\frac{1}{2}xe^{x}-\frac{1}{2}+\frac{1}{2}\int_{0}^{1}tg(t)dt$$

10

b. Solve the integral equation :

$$g(x) = 1 + \lambda \int_0^x e^{3(x-t)} g(t)dt$$ For what value of λ does the solution not exist? 8+2

UNIT-V

9a.Using Laplace transform ,Solve the integral equation:-

$$g(x) = e^{-x} - 2\int_0^x \cos(x-t) g(t)dt$$

10

b. Using Fredholm theory ,Solve the integral equation:-

$$g(x) = e^{x} + \lambda \int_0^{10} xt g(t)dt$$

10

10. Using Hilbert - Schmidt theory ,Solve the integral equation:-

$$g(x) = 1 + \lambda \int_0^{\pi} \cos(x+t) g(t)dt$$

20