Biyani Girls college ,Jaipur

Model Paper-A (M.Sc. Final)

Subject:Mathematics

Paper : Advance Numerical Analysis(XII)

Max Marks: 100 **Max Time: 2:30 hrs**

Attempt any five questions in all selecting at least one question from each unit.

UNIT-I

1.**a.** The multiple roots ξ of multiplicity two of the equation f(x)=0 is to be determined. We consider the multipoint method :-

$x_{k+1} = x_k - \frac{1}{2}\frac{(f(x_k + 2f(x_k)/f^{'}(x_k))}{f^{'}(x_k)}$ Show that the iteration method has third order rate of convergence. Hence solve the equation $9x^4 + 30x^3 + 34x^2 + 30x + 25 = 0, with\ x_0 = -1.4$

Correct to three decimal places. 10

b. Apply Aitken's Δ^2-method to find a root of the equation:- $\sin^2 x = x^2 - 1$ 10

2a. Discuss the Newton-Raphson method for a system of two non-linear equations in two unknowns. 10

b. Find a real solution of the equation: $x^3 = y + 100$; $y^3 = x + 150$ by general iteration method for a system of two non-linear equations in two unknowns. 10

UNIT-II

3a. Using Bairstow's method obtain the quadratic factor of the following equation:-

(Perform two iterations) $x^4 - 8x^3 + 39x^2 - 62x + 50 = 0$ with (p,q) =(0,0) 10

b.Solve the equation :- $x^3 - 5x^2 - 17x + 20 = 0$ by Graffee's method (squaring three times) 10

4a. Using Doolittle's Method ,Solve the following system of equations:-

2x+3y+z=9 ; x+2y+3z=6 ; 3x+y+2z=8 10

b. Solve the System of equations:- 2x-13y-3z=49 ; 5x-6y+17z=25 ; 11x+2y-4z= -31 using relaxation method. 10

UNIT-III

5a. What is power method for producing the dominant eigenvalue and eigenvector of a matrix?Apply this method to find the dominant eigenvalue and eigenvector of the matrix: $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$. 10

b. Use Jacobi's method to find all the eigenvalues and eigenvectors of matrix:- $\begin{bmatrix} 1 & \sqrt{3} & 4 \\ \sqrt{3} & 4 & \sqrt{3} \\ 4 & \sqrt{3} & 1 \end{bmatrix}$ **10**

6a. Obtain a linear and quadratic polynomial appoximaition to the function : $f(x) = x^3$ on the interval [0,1] using the least square approximation w.r.t. weight function W(x)=1 . 10

b. Prove the following recurrence relation for the Chebyshev polynomial of the first kind:

$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ Using the Chebyshev polynomial ,obtain the least squares approximation of second degree for $f(x) = x^4$ on the interval [-1,1]. 10

UNIT-IV

7a. Use fourth order runge Kutta method to solve the following initial value problem:-

$\frac{dy}{dt} = \frac{t}{2y},\ t \in [1,1.2]$ $y(1) = 2$ Compare your computed solution with the exact solution. 10

b. Derive Millen's method to solve an initial value problem:-

$\frac{dy}{dt} = f(t, y),\ t \in [t_0, b]$ $y(t_0) = y_0$ 10

8a. Use Adsoms-Moulton method to compute the solution of the initial value problem:-

$\frac{dy}{dt} = y - t^2,\ t \in [0,1]$ $y(0) = 1$ 10

b.Compute y(0.05) by Rungr-Kutta method where y(t) is the solution of the following initial value problem:- $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = 0$ $y(0) = 0, y^{'}(0) = 1$ 10

UNIT-V

9a. Disscuss finite –difference method of order two for the following boundary value problem

$$\frac{d^2y}{dx^2} = f(x, y, \frac{dy}{dx}); \quad x \in [a,b] \, with the boudary conditions \, y(a) = A \quad y(b) = B$$ **10**

b. Find the solution of the following boundary value problem

$\frac{d^2y}{dx^2} = y + x \quad x \in [0,1] \, with the boudary conditions \, y(0) = 0 \; , y(1) = 0$ with h=0.25 using

Numerov method. 10

10a. Solve the boundary value problem

$\frac{d^4y}{dx^4} = 1 \; with the boudary conditions \, y(0) = y'(0) = y(1) = y'(1) = 0$ with h=0.25 using second order difference method. 10

b. Solve the boundary value problem $\frac{d^2y}{dx^2} = y \;\; with the boudary conditions \, y(0) = 0 \quad y(1) = 1$

by Shooting method. 10