# Biyani Girls college ,Jaipur

**Model Paper-A (M.Sc. I)**

**Subject:Mathematics**

**Paper : I(Abstract Algebra)**

**Max Marks:100** **Max Time: 2:30 hrs**

**Attempt any five questions in all selecting atleast one question from each unit.**

## UNIT-I

1.a. Let $G_1$- and $G_2$ be groups then show that 10

$(i) G_1 X G_2 \cong G_2 X G_1$

(ii) Then the subsets $\hat{G}_1 = \{(a_1, e_2) : a_1 \in G_1\}$ and $\hat{G}_2 = \{(e_1, a_2) : a_2 \in G_2\}$ are normal subgroup of $G_1$ and $G_2$ respectively . $\hat{G}_1$ is isomorphic to $G_1$ and $\hat{G}_2$ is isomorphic to $G_2$ .

b. Let G be a group and suppose G is the internal direct product of subgroups $H_1, H_2 \ldots\ldots\ldots H_{n.}$ Let

$G^n = H_1 X H_2 X \ldots\ldots\ldots X H_{n.}$. Show that G and $G^n$ are isomorphic. 10

2a. Let G be a group and H is a subgroup of G .If H is normal subgroup of G and G/H is abelian,then show that $G' \subset H. Conversely\, if\ G' \subset H$ ,then show that H is normal Subgroup of G. 10

b. Show that any two Subnormal series of a group G have subnormal refinements that are equivalent .10

## UNIT-II

3a. Prove that the ring of all Gaussian integer Z[i] is a Euclidean Ring. 10

b. Let N,K be the Sub modules of the R – Module M . Then Prove $\frac{(N+K)}{K} \cong \frac{N}{N \cap K}$ 10

4a. Let V and V' be vector spaces over the same field F s.t. the dimension of V is finite .Let $t : V \to V'$ be a linear transformation ,then prove that : dim V = rank(t) + nullity (t) 10

b. Let V be n-dimensional vector spaces over a field F. Let B={$b_1, b_2, \ldots\ldots b_n$}be a basis of V then prove that the dual space $V^*$ of V has basis $B^*$={$f_1, f_2, \ldots\ldots f_n$} such that

$f_i(b_j) = \delta_{ij} \quad : i, j = 1,2,\ldots\ldots .n \ where \ \delta_{ij} \in F$ is Kronecker Delta 10

## UNIT-III

5a. Let $\frac{K}{F}$ be a field extension .Then prove that an element $a \in K$ is algebraic over F iff F(a) is finite extension of F. 10

b. Prove that an irreducible polynomial f(x) over a field F of Characteristic p>0 is inseparable iff f(x) is a polynomial in $x^p$. 10

6a. Let G be a finite group of automorphism of a field K. Let F be the fixed field of G that is :

$F = \{x \in K | \phi(x) = x, \ forall \, \phi \in G\}$.Then prove that K is Galois extension of F with $G(K/F) = G.$ 10

b. Prove that the general polynomial equation of degree n is not solvable by radicals for $n \geq 5.$ 10

## UNIT-IV

7a. If a square matrix A of order n, over a field F, has n distinct eigenvalues $\lambda_1, \lambda_2 \ldots\ldots, \lambda_n$ then there is an invertible matrix P such that $P^{-1}AP = diag(\lambda_1, \lambda_2 \ldots\ldots, \lambda_n)$ 10

b. Let V be an n-dimensional vector spaces over a field F and $V'$ be an m-dimensional vector space over F. Then show that : $Hom(V, V') \cong M_{m \times n}(F)$. 10

8a. Let $f : (F^n)^n \to F$ be a multilinear alternating form. Then show that for any n x n matrix $A = [a_{ij}] \, over \, F. \ f(A) = \det(A) \, f(I)$ where I is n x n identity matrix. 10

b. Let A be an n x n identity matrix over a field F .

$\det(A - \lambda I) = f(\lambda) = a_n\lambda^n + a_{n-1}\lambda^{n-1} + \ldots\ldots\ldots \ldots + a_1\lambda + a_0$ be a characterstic polynomial of A. Then show that: $f(A) = a_n A^n + a_{n-1}A^{n-1} + \ldots\ldots\ldots \ldots + a_1 A + a_0 I = 0$ 10

## UNIT-V

9a. Let V be inner product space .Then for all $u, v \in V \text{ and } \alpha \in R$ Prove that

$(i) \|\alpha u\| = |\alpha| \|u\|$ $(ii)$ $\big| \|u\| - \|v\| \big| \le \|u - v\|$

b. Let V be an inner product space and A={$v_1$,$v_2$,………..$v_n$} be an orthonormal set in V .Then prove that any vector $v \in V$ the vector : $u = v - \sum_{i=1}^{n} v_i < v, v_j >$ is orthogonal to each $v_j$, j=1,2,3,…..n and consequently to the subspace generated by the orthonormal set A.

10a. Let V be a finite dimensional inner product space and W be its any subspace .Then prove that V is the direct sum of $W \text{ and } W^{\perp}$ ie. $V = W \oplus W^{\perp} \text{ when } W^{\perp}$ is orthogonal complement of W.

b. Let V and V' be inner product spaces.Then prove that a linear transformation $t : V \to V'$ is orthogonal iff : $\|t(u)\| = \|u\| \text{ for all } u \in V$ .