Biyani Girls college ,Jaipur

Model Paper-B (M.Sc. I)

Subject:Mathematics

Paper : I(Abstract Algebra)

Max Marks:100 **Max Time: 2:30 hrs**

Attempt any five questions in all selecting atleast one question from each unit.

UNIT-I

1.a Let H be a subgroup of G and K be a normal subgroup of G then show that : $\frac{HK}{K} \cong \frac{H}{H \cap K}$

Further suppose K_1 is some other normal subgroup of G s.t. $H \cap K = H \cap K_1$ then $\frac{HK}{K} \cong \frac{HK_1}{K_1}$ .10

b. A group G is an internal direct product of subgroup $H_1, H_2 \ldots\ldots H_n$ iff

(i) G = $H_1, H_2 \ldots\ldots H_n$ (ii) $H_1, H_2 \ldots\ldots H_n$ are all normal subgroup of G and

$(iii)\, H_i \cap (H_1 H_2 \ldots\ldots H_{i-1} H_{i+1} \ldots\ldots Hn) = \{e\}$ 10

2a. Let R be an Euclidian ring and $a_o \in R$ .Then prove that the principal ideal $A = < a_o > \ of\ R$ is maximal ideal of R Iff a_o is a prime element of R. 10

b. Let M be an R- module and let N and K be sub module of M with $N \subseteq K$ then prove that

$(M/N)|(K/N) \cong M/K.$ 10

UNIT-II

3a. Let V be a finite dimensional vector space over a field F and W be any subspace of V. If A(W) denotes annihilator of W ,then prove that: dim A(W) = dim V – dim W. 10

b. Let M be an R-Module .Then show that M is the direct sum of its sub modules $N_1, N_2 \ldots\ldots\ldots N_K$ Iff

$(i) M = N_1 + N_2 + \ldots\ldots\ldots\ldots\ldots + N_K$

$(ii)\, N_i \cap (N_1 + N_2 + \ldots\ldots\ldots + N_{i-1} + N_{i+1} \ldots\ldots\ldots + N_K) = \{\bar{0}\}$ for all $i = 1, 2 \ldots\ldots k$ 10

4a. Let V and V' be vector spaces over a field F. Let $B=\{b_1, b_2, \ldots\ldots\ldots b_n\}$ be a basis of V and $B' = \{b_1', b_2', \ldots\ldots, b_n'\}$ be a set of vectors in V'.Let $t: V \to V'$ be a linear transformation s.t.

$t(b_i) = b_i', i = 1, 2 \ldots\ldots n$ *then show that t is an isomorphism iff the set* B' *is a basis of* B'. 10

b. Let V and V' be finite dimensional vector space over a field F and $t: V \to V'$ be a linear transformation then prove that ker (t^*) = A(t(V)), Hence show that t and t^* have the same rank where t^* is the dual map of t and A(t(V))is the annihilator of the image sub-space t(V). 10

UNIT-III

5a. Prove that every finite extension of a field is an algebraic extension. But the converse is not necessarily true. **10**

b. Define normal field extension .Let K be a normal extension of afield F and L is an intermediate field such that $F \subset L \subset K$,then prove that K is also a normal extension of L. 10

6a. Prove that the order of the Galois group $G(K/F)$ is equal to the degree of K over F ie.

$O[G(K/F)] = [K:F]$. 10

b. Show that the Galois Group of : $f(x) = x^4 + 1 \in Q[x]$ is the Klein's four group. 10

UNIT-IV

7a. A square matrix A over a field F is invertible iff it can be written as a product of elementary matrices. 10

b. Let t be a linear transformation of a finite dimensional vector space to itself .Then Show that the matrix A representing t is diagonal with the eigen values of t as diagonal entries iff A is relative to a basis of V consisting of eigenvectors of t. 10

8a. An nxn square matrix A is an invertible matrix over F iff $\text{Det}(A) \neq 0$ 10

b. Similar matrices have the same characteristic polynomial and hence the same eigen values. 10

UNIT-V

9a. Let V be an inner product space and $u, v \in V \; s.t. \; \|u\| = \|v\| = 1$ then show that : $|<u, v>| \leq 1$ Hence or otherwise show that for arbitrary vectors $u, v \in V$ we have $|<u, v>| \leq \|u\| \|v\|$ 10

b. If $\{u_1, u_2, \ldots\ldots\ldots u_n\}$ is any finite orthonormal set in an inner product space V and v is any vector in V ,then show that $\sum_{i=1}^{n} |<v, u_i>|^2 \leq \|v\|^2$ 10

10a. State and prove principle axis thorem. 10

b. Let V and V' be inner product spaces. Then a linear transformation $t : V \rightarrow V'$ is orthogonal iff $\|t(u)\| = \|u\| \; for all \, u \in V.$ 10