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M.Sc (P) Mathematics
Paper-IV , set A
Differentinl Geometry

Time: 2:30 **Maximum Marks: 100**

Unit I

1. (a) Determine a,h,b so that the paraboloid $2z=ax^2+2hxy+by^2$ may have the closest contact at the origen with the curve $x= t^3-2t^2+1$, $y= t^3-1$, $z=t^2-2t+1$. Also find the order of contact.

(b) Define osculating circle, show that the tangent to the locus of the centre of curvature lies in the normal plane of the original curve and is inclined at an angle

$$\tan^{-1}\left(\frac{\rho\tau}{\rho'}\right)$$

2. (a) Find osculating plane, curvature and torsion at any point θ of the curve

$$x = a\cos 2\theta, y = a\sin 2\theta \ \ and \ \ z = 2a\sin\theta.$$

(b) Prove that the radii of curvature and torsion of the spherical indicatrix of the binomials are:

$$\frac{\rho}{\sqrt{\sigma^2+\rho^2}}, \ \ and \ \ \frac{\rho^2+\sigma^2}{\sigma(\rho'\sigma-\sigma'\rho)}$$

Unit II

3. (a) Show that the edge of regression of the envelope of the plane

$\frac{x}{a+\lambda}+\frac{y}{b+\lambda}+\frac{z}{c+\lambda}=1$ is the given by $x=\frac{(a+\lambda)^3}{(c-a)(b-a)}$,

$x=\frac{(b+\lambda)^3}{(c-b)(a-b)}$, $x=\frac{(c+\lambda)^3}{(a-c)(b-c)}$,

(b) show that the developable which passes through the curves

$z=0, y^2=4ax \ ; and \ x=0, y^2=4bz$ is the cylinder $y^2 = 4ax + 4bz$.

4. (a) Show that on a right helicoids, the family of curves orthogonal to the curves ucos v = constant is the family $(u^2+a^2)\sin^2 v =$ constant.

(b) obtain the following Serret Frenet formula $\frac{d\hat{t}}{ds}=\kappa\hat{n}$, $\frac{d\hat{b}}{ds}=-\tau\hat{n}$

And $\frac{d\hat{n}}{ds} = \tau\hat{b} - \kappa\hat{t}$.

Unit-III

5. (a) if the first ,second and third fundamental forms of a surface are denoted by I, II, III respectively prove that $\kappa I - 2\mu II + III = 0$

(b) Prove that the necessary and sufficient condition that a curve on a surface be line of curvature is that $d\hat{N} - \kappa dr = 0$ at each of its points.

6. (a) Prove that the necessary and sufficient then prove that parametric curves be lines of curvature are F=0,M=0 and EN-GL‡0.

(b) find the principal radii at the origin og the surface $2z = 5x^2+4xy+2y^2$. Find also the radius of curvature of the section x=y.

Unit-IV

7 (a) Find that the necessary and sufficient condition for a curve u =u(t), v = v(t) to be geodesic on a surface r= r(u,v)is $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$

(b) Find the geodesic on a surface of revolution $x = u\cos\theta, y = u\sin\theta, z = f(u)$

8. (a) Show that the metric of a Euelidean space referred to spherical coordinates is given by $ds^2 = (dr)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2$ determine the metric tensorand conjugate metric tensor

(b) Prove that $A^{ij}_{j} = \frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}A^{ij})}{\partial x^j} + A^{jk}\begin{Bmatrix} i \\ j \quad k \end{Bmatrix}$. Show that the last term Vanish if A^{ij} is skew symmetric.

Unit -V

9. (a) Surface of a sphere is a two dimensional Riemannian space . compute the christoffel symbols.

(b) Define covariant derivative of a tensor. If A^{ijk} is a skew symmetric tensor show that $\frac{1}{\sqrt{g}}\frac{\partial(\sqrt{g}A^{ijk})}{\partial x^k}$ is a tensor

10 .(a) Define Einstein tensor . Prove that:

(i) the divergence of Einstein tensor vanishes.

(ii) An Einstein space V_N (N>2) has constant curvature.

(b) Define geodesic coordinate system . Show that it is always possible to choose n coordinate system which is geodesic in nature at particular point P_0.