Biyani's Group of Colleges

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M.Sc (P) Mathematics
Paper-IV , set A
Differentinl Geometry

Time: 2:30 **Maximum Marks: 100**

Unit I

1. (a) Find the equation of osculating plane of the curve given by

   $r = (a\sin t + b\cos t, a\cos t + b\sin t, c\sin 2t)$. ·

   (b) if the tangent and the binomial at a point of a curve make angle θ, ϕ

   Respectively with a fixed direction prove that $\frac{\sin\theta}{\sin\phi}\frac{d\theta}{d\phi} = \pm\frac{\kappa}{\tau}$

2. (a) Find the equations of the principal normal and of the osculating plane at any point of the curve given by the equation

   $x = 4a\cos^3\theta, y = 4a\sin^3\theta \ \ and \ \ z = 3c\sin 2\theta.$

   (b) Find the spherical indicatrix of the tangent, binomials and principal normal of the circular Helix $x = a\cos\theta, y = a\sin\theta \ \ and \ \ z = a\theta\cot\alpha.$

Unit II

3. (a) Prove that the metric of a surface is invariant under parametric transformation.

   (b) Derive the Weingraten equations .

4. (a) Find the fundamental magnitudes of general surface of revolution .

   (b) show that the parametric curves on the sphere

   $x = a\sin u\cos v, y = a\sin u\sin v \ \ and \ \ z = a\cos u.$

Unit-III

5. (a) Derive the radius of curvature of any normal section at an umbilic on the surface Z= f(x,y).

   (b) Find the principal sections and the principal curvatures of the surface x = a(u+v), Y=b(u-v), z = uv.

6. Derive the principal radii and lines of curvature through a point of the surface Z= f(x,y).

Unit-IV

7 (a) Find that the necessary and sufficient condition for a curve u =u(t), v = v(t) to be geodesic on a surface r= r(u,v)is $U\frac{\partial T}{\partial \dot{v}} - V\frac{\partial T}{\partial \dot{u}} = 0$

(b) Find the geodesic on a surface of revolution $x = u\cos\theta, y = u\sin\theta, z = f(u)$

8. (a) if A^i is an arbitrary contravariant vector and $C_{ij}A^iA^j$ is an invariant prove that $C_{ij} + C_{ji}$ is A covariant tensor of second order.

(b) State and prove that quotient law of tensors

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Unit -V

9. (a) define Christoffel symbols of the first and second kind prove that

$$\frac{\partial g_{ik}}{\partial \dot{x}^j} = [ij,k] + [kj,i]$$

(b) if $A_{ij} = B_{i,j} - B_{j,i}$ *then prove that* $A_{ij,k} + A_{jk,i} + A_{ki,j} = 0$

10 .(a) Find the metric of a Euclidean space referred to spherical coordinates.

(b) Define geodesic coordinate system . Show that it is always possible to choose n coordinate system which is geodesic in nature at particular point P_0.