M.Sc (P) Mathematics
Paper-III , set A
Differential equation

Time: 2:30 **Maximum Marks: 100**

Unit I

1. (a) solve $2y\frac{d^3y}{dx^3}+2(y+3\frac{dy}{dx})\frac{d^2y}{dx^2}+(\frac{dy}{dx})^2=2$

   (b) Solve $\frac{d^2y}{dx^2}+\frac{dy}{dx}+(\frac{dy}{dx})^3=0$.

2. (a) Solve $(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$

   (b) Solve $r = a^2 t$ by Monge's method.

Unit 2

3. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\left(\frac{\partial^2 z}{\partial x^2}\right)$ to canonical form and classify character.

   (b) Solve the two dimensional heat conduction equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\frac{1}{k}\left(\frac{\partial z}{\partial t}\right)$

   by the method of separation of variables.

4. (a) Find the eigenvalues and eigenfunction of the Sturm Liouville problem:

   $$\frac{d}{dx}\left[x\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0, y(1)=0\, and\, y(e^{\pi})=0$$

   (b) State orthogonal property of eigenfunctions prove that the eigenvalues of Sturm Liouville system are real

Unit-3

5. (a) Establish Euler lagrange differential equation for an extremal of a function of the form $I[y(x)]=\int_{x_0}^{x_1} f(x,y,y^{'})dx;$

   (b) Determine the extremal of the functional $I[y(x)]=\int_{-1}^{1}\left(\frac{1}{2}\mu y^{''2}+\rho y\right)dx;$

That satisfied the boundary conditions $y(-1)=0, y'(-1)=0, y(1)=0, y'(1)=0$

6. (a) Find the solution in series of the following differential equation

$$(x-x^2)y_2+(1-5x)\frac{dy}{dx}-4y=0$$

(b) Show that the shortest line between any two points on a cylinder is a helix.

Unit-4

7. (a) Write the integral representation for ${}_2F_1$ with the condition of validity and obtain from it the following: (i) Gauss Theorem (ii) Kummers Theorem (iii) vander modes theorem.

(b) Prove That $(1-x)F=F(-a)-\left(\frac{c-b}{c}\right)xF(c+)$ whrer F stands for gauss hypergeometric function ${}_2F_1$(a,b,c,x).

8. (a) Prove that $\int_{-1}^{1} x^2 p_{n+1}(x)p_{n-1}(x)dx=\frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}$.

(b) Establish: $nQ'_{n+1}(x)+(n+1)Q'_{n-1}(x)=(2n+1)xQ_n^{1}(x)$

Unit -5

9. (a) if n is an integer , show that:

$$J_n(x)=\frac{1}{x}\int_0^{\pi}\cos(n\phi-x\sin\phi)d\phi$$

(b) Prove that $P_n(x)=\frac{2}{n!\sqrt{\pi}}\int_0^{\infty}e^{-t^2}t^nH_n(xt)dt$

10 .(a) prove that $L_n(x)=\frac{e^x}{n!}\frac{d^n(x^ne^{-x})}{dx^n}$

(b) state and prove orthogonal property of associated Laguerre’s polynomoial.