Biyani Girls college ,Jaipur

Model Paper-A (M.Sc. I)

Subject:Mathematics

Paper : V (Machenics)

Max Marks: 100 **Max Time: 2:30 hrs**

Attempt any five questions in all selecting atleast one question from each unit.

UNIT-I

1.a. A rough uniform board of mass m and length 2a , rests on a smooth horizontal plane and a man of mass M walks on it from one end to the other .Find the distance through which the board moves in this time. 10

b. A uniform wire, in the form of an arc of a circle of a given radius is swinging about a horizontal axis through the middle point of the arc perpendicular to the plane of the arc. Show that the time of a small oscillation is independent of the length of an equivalent simple pendulum is equal to the diameter of the circle. 10

2a. A sphere of radius ‘a’ is suspended by a fire wire from a fixed point at a distance l from its center show that the time of a small oscillation is given by:-

$2\pi\sqrt{\frac{5l^2+2a^2}{5\lg}}\left[1+\frac{1}{4}\sin^2\frac{\alpha}{2}\right]$ where α represents the amplitude of the vibration. 10

b. A hollow cylinder ,of radius ‘a’ , is fixed with its axis horizontal inside it moves a solid cylinder of radius ‘b’ whose angular velocity in its lowest position is Ω .If the friction between the cylinders be sufficienr to prevent any sliding prove that the small cylinder will just go round the inner surface if

$\Omega=\sqrt{\frac{11}{3}g/(a-b)}$. 10

UNIT-II

3a. A solid cube is in motion about an angular point which is fixed.If there are no external forces and $\omega_1, \omega_2, \omega_3$ are the angular velocities about that:

$\omega_1 + \omega_2 + \omega_3 = con\tan t \quad and \quad \omega_1^2 + \omega_2^2 + \omega_3^2 = cons\tan t$ 10

b. Deduce the Euler's dynamical equations from Lagrange's equation of motion. 10

4a. State and prove the principle of conservation of momentum for finite forces. 10

b. Define angular momentum .State and prove the principle of conservation of angular momentum under impulsive forces . 10

UNIT-III

5a. Obtain Lagrange's Equation of motion in generalized coordinates for a holonomic dynamical system. 10

b. Three equal uniform rods AB,BC,CD are freely jointed at B and C and the ends A and D are fastened to smooth fixed pivots whose distance apart is equal to the length of either rod.The frame being atrest in the form of the square ,a blow I is given perpendicular to AB at its middle point and in the plane of the square .Show that the energy set up is $\frac{3l^2}{40m}$ where m is the mass of each rod. 10

6a. Derive Largrange's equations under impulsive forces motion. 10

b. Derive equation of motion of a top. 10

UNIT-IV

7a. Determine the equation of continuity in Cartesian coordinates. 10

b. Show that variable ellipsoid :-

$\frac{x^2}{a^2k^2t^4} + kt^2\left[\frac{y^2}{b^2} + \frac{z^2}{c^2}\right] = 1$.is a possible form of the boundary surface of a liquid at time t. 10

8a. Find the image of source with respect to a circle in two Dimensions. 10

b. Explain the complex potential of source. 10

UNIT-V

9a. A mass of liquid surrounds solid sphere of radius ‘a’ and its outer surface which is a concentric sphere of radius b is subjected to a given pressure Π,no other force being in action on the liquid .The solid sphere suddenly shrinks into a concentric sphere. Determine the subsequent motion. 10

b. An infinite mass of fluid is acted on by a force $\mu r^{-3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere r = c in it, show that the cavity will be filled up after an interval of time $\sqrt{\frac{2}{5\mu}} c^{5/4}$

10a. Use the method of images to prove that if there be a source ‘m’ at the point z_0 in a fluid bounded by the lines $\theta = 0$ and $\theta = \frac{\pi}{3}$, the solution is:-

$\phi + i\varphi = -m\log\left[(z^3 - z_0^{\ 3})(z^3 - z'^{\ 3}_0)\right]$ where $z_0 = x_0 + iy_0$ and $z'_0 = x_0 - iy_0$ 10

b. In the case of two dimensional fluid motion produced by a source of strength ‘m’ placed at a point ‘S’ outside a rigid circular disc of radius ‘a’ whose centre is ‘O’, show that the velocity of slip of the fluid in contact with the disc is greatest at the points where the lines joining ‘S’ to the ends of the diameter at the right angles to OS cut the circle and prove that its magnitude at these points is $\frac{2m.OS}{(OS^2 - a^2)}$. 10