Biyani Girls college ,Jaipur

Model Paper-B (M.Sc. I)

Subject:Mathematics

Paper : V (Machenics)

Max Marks: 100 **Max Time: 2:30 hrs**

Attempt any five questions in all selecting atleast one question from each unit

UNIT-I

1.a. State and prove D'Alembert 's Principle for the motion of rigid body. 10

b. A rod of length 2a is suspended by a string of length l, attached to one end if the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively ,show that : $\frac{3l}{a} = \frac{(4\tan\theta - 3\tan\phi)\sin\phi}{(\tan\phi - \tan\theta)\sin\theta}$. 10

2a. Find the center of precession of a triangle ABC which is free to move about its side BC. 10

b. A uniform solid cylinder is placed with its axis horizontal on a plane ,whose inclination to the horizon is α. Show that the least coefficient of friction between it and the plane ,so that it may roll and not side ,is 1/3 tanα. 10

UNIT-II

3a. Show that the K.E. and angular momentum of a body are constants during force free motion about a fixed point . 10

b. A rigid body moves under no forces about a point O, the principal moments of inertia at O being 6A,3A and A.Initially the angular velocity of body has components about the principal axis .Show that at any later time $\omega_2 = -\sqrt{5n}\tanh\sqrt{5n}t$ and ultimately the body rotates about the mean axis. 10

4a. A uniform square lamina of mass M and side 2a , is moving freely about a diagonal with uniform angular velocity ω when one of the corners not in that diagonal become fixed.Show that the new angular velocity is 1/7 ω and the impulse of the force on the fixed point is $\frac{\sqrt{2}}{7}Ma\omega$. 10

b. A small insect moves along a uniform bar , the mass equal to itself and of a length 2a ,the ends of which are constrained to remain on the circumference to remain on the circumference of a fixed circle ,whose radius is $2a/\sqrt{3}$ .If the insect starts from the middle point of the bar and moves along the bar with relative velocity V, Show that the bar in time t will turn through an angle $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{Vt}{a}\right)$. 10

UNIT-III

5a. Explain the terms – Degree of freedom ,generalized coordinates .Disscuss the classification of constraitns. 10

b.A uniform bar of length 2a is hung from a fixed point by a string of length b fastened to one end of the bar .Show that when the system makes small normal oscillations in a vertical plane ,the length l of the simple equivalent pendulum is a root of the quadratic $l^2-\left(\frac{4a}{3}+b\right)l+\frac{ab}{3}=0$ 10

6a. A Circular disc of radius 'a' has a thin rod pushed through its center perpendicular to its plane the length of the rod being equal to the radius of the disc. Show that the system can not spin with the rod vertical unless the angular velocity is greater than $\sqrt{\frac{20g}{a}}$. 10

b. A sphere rolls down a rough inclined plane .I f x be the distance of the point of contact of the sphere from a fixed point on the sphere from a fixed point on the plane, determine its acceleration using Hamilton 's equations of motion. 10

UNIT-IV

7a. Explain the concepts of streamlines ,path lines and streamlines .Show that at all points of the fields of flow ,the equipotentials are cut orthogonally by the streamlines . 10

b. If the motion of an incompressible ideal fluid is of the potential kind ,find the equation of continuity.

8a. A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis .Show that the equation of continuity is $\frac{\partial\rho}{\partial t}+\frac{\partial(\rho\omega)}{\partial\theta}=0$ where ω is angular velocity of the particle whose azimuthal angle is θ at time t. 10

b. Show that if the velocity potential of an irrotational fluid motion is equal to :-

$A(x^2 + y^2 + z^2)^{-3/2} z \tan^{-1} y/x$ the lines of flow will be on the series of surfaces:

$(x^2 + y^2 + z^2) = c^{2/3}(x^2 + y^2)^{2/3}$. 10

UNIT-V

9a. Derive the Euler's dynamical equations of motion in Cartesian coordinates. 10

b. Stream is rushing from a boiler through a conical pipe the diameters of the ends od which are D and d. If V and v be the corresponding velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone , prove that $\frac{v}{V} = \frac{D^2}{d^2}\left[e^{(v^2 - V^2)/2k}\right]$ where k is the pressure divided by the density and supposed constant. 10

10a. Prove that the image of a source w.r.t. to a straight line is an equal source placed at equal distance on the other side of the line. 10

b. State and drive Helmholtz equations. 10