M.Sc. (Prev.) Mathematics

Model Paper - A

Paper -I (Real Analysis and Topology)

Time allowed : 3 Hrs Max. Marks : 100

Note : Attempt any five question in all. Select at least one from each unit. Each question carry equal marks

UNIT -I

Q.1(a) Define outer measure of a set. Prove that the outer measure of an interval is its length.

(b) Prove that every interval is measurable

Q.2(a) Define equivalent functions. If f is a measurable function defined on a measurable set E and if a function g is equivalent to f, then show that g is also measurable function on E

(b) Show that a sequence of measurable functions converging to a measurable function almost everywhere converges to the some function in measure.

UNIT -II

Q.3(a) Let f be a continuous real valued function defined on [0,1]. Then prove that for every e>0, thee exists a polynomial function $p(x), s, t$

$f(x) - p(x)/< E$ for all $x \in [0,1]$

(b) Let f be a bounded measurable function on a set E and let $f(x) \geqslant 0\ a.e$ on E, If $\int_E F(x)\,dx = 0$, then prove that $f(x) = 0\ a.e.$on E.

or

Q.4(a) Let f be summable function on a set E. Then prove that for a given $\in > 0$, there exists a $\delta > 0$ s.t

$$\int_\in |f(x)|\,dx <\in$$

(b) Prove that the series $\sum_{i=1}^{\infty}$ of pair wise orthogonal elements in L_2 is convergent iff the series of real numbers $\sum_{i=1}^{\infty} \|f_i\|^2$

UNIT -III

Q.5(a) Prove that L_p-space is complete.

(b) Let $1 < p < \infty$ and q be a non negative real number conjugate to p. if $f \in L^p$ and $g \in L^q$ then prove that $\int_{\in} f(x)g(x)dx \leq \|f\|_p \, \|g\|_q$

Q.6 (a) Show that a subset A of topological space (x, τ) is open iff A is a neighborhood of each point in it.

(b) Let (x, τ) be a topological space and $Y \subset X$. Then prove that the collection $\tau_y = \{G \cap Y / G \in \tau\}$ is topology on Y.

UNIT -IV

Q.7(a) If (x, τ_1) and (x, τ_2) are any two topological spaces then prove that the mapping f: X → Y is continuous on X iff for every subset $A \subset X, f(\bar{A}) \subset \overline{f(A)}$, where $\bar{A}$ denotes the closure of A.

(b) Define T_1 space. Prove that a topological space (x, τ) is a T_1 space. Iff every singleton subset $\{x\}$ of X is a closed set.

Or

Q.8(a) Prove that a topological space (x, τ) is a regular space iff given a point p in X with $p \in G$, $G \in \tau$, there exist an open set V such that $p \in V \subset \bar{V}$ where $\bar{V}$ denotes the closure of V

(b) Prove that the product of two Hausdroff space is Hausdroff space .

UNIT -V

Q.9(a) Prove that a compact Hausdroff space is normal

(b) Define a locally compact space. Prove that every open continuous image of a locally compact space is locally compact.

Or

Q.10(a) Prove that the union of any family of connected sets having a non empty intersection is connected.

(b) Prove that every open subspace of a locally connected topological space is locally connected

M.Sc. (Prev.) Mathematics

Model Paper - B

Paper -I (Real Analysis and Topology)

Time allowed : 3 Hrs Max. Marks : 100

UNIT -I

Q.1(a) Define an algebra of set. Show that for any collection α of subset of a set X, there is a smallest algebra A* containing α.

(b) Let A be any set and $\{E_i / i \in \underline{n}\}$ be a finite class of disjoint measurable sets then prove that

$$m^* \left\{ A \cap \left(\bigcup_{i=1}^{n} E_i \right) \right\} = \sum_{i=1}^{n} m^*(A \cap E_i)$$

Or

Q.2(a) Let $\langle f_n \rangle$ be a sequence of measurable function which converges in measure to the function f then prove that their exist a subsequence of $\langle f_n \rangle$ which also converges two f almost everywhere.

(b) Let f be a measurable function finite almost everywhere defined on a closed interval $E = [a, b]$. then prove that for all $\sigma > 0$ and $\epsilon > 0$, there exist a continuous function ϕ defined on E such that

$$m\{x \in E / |f(x) - \phi(x)| \geq \sigma\} < \epsilon$$

UNIT -II

Q.3(a) Define the lebesgue integral for a bounded measurable function. Prove that every bounded and measurable function defined on a measurable set E is lebesgue integrable on E.

(b) Let $\langle f_n \rangle$ be a sequence of measurable function Ψ defined on a measurable set E such that $|f_n(x)| < \Psi(x)$ for all $x \in E$ and for all $\in N$ where $\Psi(x)$ is an integrable function over E. If $\langle f_n \rangle$ converges in measure to the measureable function f on E then prove that

$$\lim_{n\to\infty} \int_E f_n(x) dx = \int_E f(x)\ dx$$

Or

Q.4(a) Show that space of square summable function L_2 is a normed linear space.

(b) Prove that the Fourier series of any function $f \in L_2$ converges in norm prove that it converges to f iff

$$\|f\|^2 = \sum_{i=1}^{\infty} a_i^2$$

Where a_i is the fourier coefficient of the function f.

UNIT- III

Q.5(a) Define L^p space. Prove that the L^p space is a linear space

(b) Prove that a normed linear space L^p space $1 \leq p < \infty$ is complete and hence a banach space.

Or

Q.6(a) Define accumulation point. Prove that a subset A of a topological space $(x, \tau)$is closed iff $A' \subset A$, A' being the derived set of A.

(b) Let (x, τ) be a topological space and let $A, B \subset$ X. Then prove that

(i) $A \subset B \Rightarrow \bar{A} \subset \bar{B}$ (ii) $\overline{A \cup B} = \bar{A} \cup \bar{B}$

(iii) $\overline{A \cap B} \subset \bar{A} \cap \bar{B}$

Where $\bar{A}$ denotes the closure of the set A.

UNIT -IV

Q.7(a) Define continuous mapping. Prove that if (x, τ_1) and (y, τ_2) are two topological spaces then the mapping $f: X \to Y$ is continuous on X iff inverse image of each closed set in Y is closed in X.

(b) Define T_2 space. Prove that every T_2 space is a T_1 space but the converse is not necessarily true.

Or

Q.8(a) Prove that a topological space (x, τ) is a normal space iff for any closed set F and open set G containing F, there exist an open set V such that $F \subset V \subset \bar{V} \subset G$.

(b) Define a filter on a non empty set X and prove that the intersection of an arbitrary family of filters on X is a filter on X.

UNIT -V

Q.9(a) Define compact space and prove that every compact subset of a Hausdorff space is closed.

(b) Prove that every open continuous image of a locally compact space is locally compact.

Or

Q.10(a)Define connected space. Prove that a subspace Y of a topological space (x, τ) is connected iff for a every pair open subset A and B of X such that $Y \subset A \cup B$ and $Y \cap A, Y \cap B$ are non empty, the set $Y \cap A \cap B$ is non empty.

(b)Define a component of a topological space and prove that every component locally connected space an open set.